6.9 Wave Forces on a Body

\[ U = \omega A \]
\[ Re = \frac{U\ell}{\nu} = \frac{\omega A \ell}{\nu} \]
\[ K_e = \frac{UT}{\ell} = \frac{A\omega T}{\ell} = 2\pi \frac{A}{\ell} \]

\[ C_F = \frac{F}{\rho g A \ell^2} = f\left( \frac{A}{\lambda}, \frac{\ell}{\lambda}, Re, \frac{h}{\lambda}, \text{roughness, \ldots} \right) \]

- Wave steepness
- Diffraction parameter

Wave steepness
Diffraction parameter
6.9.1 Types of Forces

1. **Viscous forces** Form drag, viscous drag $= f(R_e, K_c, \text{roughness}, \ldots)$.
   
   - *Form drag ($C_D$)*
     
     Associated primarily with flow separation - normal stresses.

   ![Particle velocity](image1.png)

   - *Friction drag ($C_F$)*
     
     Associated with skin friction $\tau_w$, i.e., $\vec{F} \sim \iint_{\text{body (wetted surface)}} \tau_w dS$.

   ![B.L.](image2.png)
2. **Inertial forces** Froude-Krylov forces, diffraction forces, radiation forces.

Forces arising from potential flow wave theory,

\[
\mathbf{F} = \iint p\hat{n}dS, \text{ where } p = -\rho \left( \frac{\partial \phi}{\partial t} + gy + \frac{1}{2} |\nabla \phi|^2 \right)
\]

For linear theory, the velocity potential \( \phi \) and the pressure \( p \) can be decomposed to

\[
\phi = \phi_I + \phi_D + \phi_R
\]

\[
-\frac{p}{\rho} = \frac{\partial \phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t} + \frac{\partial \phi_R}{\partial t} + gy
\]

(a) **Incident wave potential**

- **Froude-Krylov Force approximation** When \( \ell << \lambda \), the incident wave field is not significantly modified by the presence of the body, therefore ignore \( \phi_D \) and \( \phi_R \). Froude-Krylov approximation:

\[
\begin{align*}
\phi & \approx \phi_I \\
p & \approx -\rho \left( \frac{\partial \phi_I}{\partial t} + gy \right)
\end{align*}
\]

\[
\Rightarrow \mathbf{F}_{FK} = \iint p_I \hat{n}dS \equiv p_I
\]

- **Mathematical approximation** After applying the divergence theorem, the \( \mathbf{F}_{FK} \) can be rewritten as \( \mathbf{F}_{FK} = -\iiint p_I \hat{n}dS = -\iiint \nabla p_I dV \).

If the body dimensions are very small comparable to the wave length, we can assume that \( \nabla p_I \) is approximately constant through the body volume \( \forall \) and ‘pull’ the \( \nabla p_I \) out of the integral. Thus, the \( \mathbf{F}_{FK} \) can be approximated as

\[
\mathbf{F}_{FK} \approx \left[ -\nabla p_I \right] \iiint_{\text{body center}} dV = \iiint_{\text{body volume}} \nabla \left[ -p_I \right] \text{ at body center}
\]

The last relation is particularly useful for small bodies of non-trivial geometry - for 13.021, that is all bodies that do not have a rectangular cross section.
(b) Diffraction and Radiation Forces

(b.1) Diffraction or scattering force When $\ell \ll \lambda$, the wave field near the body will be affected even if the body is stationary, so that no-flux B.C. is satisfied.

\[
\frac{\partial \phi}{\partial n} = 0 = \frac{\partial}{\partial n} (\phi_i + \phi_D)
\]

or \[ \frac{\partial \phi_D}{\partial n} = - \frac{\partial \phi_i}{\partial n} \leftarrow \text{given} \]

\[
\vec{F}_D = \int_{\text{body surface}} -\rho \left( \frac{\partial \phi_D}{\partial t} \right) \hat{n} dS
\]

(b.2) Radiation Force - added mass and damping coefficient Even in the absence of an incident wave, a body in motion creates waves and hence inertial wave forces.

\[
\vec{F}_R = \int_{\text{body surface}} -\rho \left( \frac{\partial \phi_R}{\partial t} \right) \hat{n} dS = -m_{ij} \ddot{U}_j - d_{ij} U_j
\]
6.9.2 Important parameters

\[ K_c = \frac{U_T}{\ell} = 2\pi \frac{A}{\ell} \]

Interrelated through maximum wave steepness

\[ \frac{A}{\lambda} \leq 0.07 \]

\[ (\frac{A}{\ell}) \left( \frac{\ell}{\lambda} \right) \leq 0.07 \]

- If \( K_c \leq 1 \): no appreciable flow separation, viscous effect confined to boundary layer (hence small), solve problem via potential theory. In addition, depending on the value of \( \frac{\ell}{\lambda} \),
  - If \( \frac{\ell}{\lambda} \ll 1 \), ignore diffraction, wave effects in radiation problem (i.e., \( d_{ij} \approx 0, m_{ij} \approx m_{ij} \) infinite fluid added mass). F-K approximation might be used, calculate \( \vec{F}_{FK} \).
  - If \( \frac{\ell}{\lambda} \gg 1/5 \), must consider wave diffraction, radiation \( (\frac{A}{\ell} \leq \frac{0.07}{\ell/\lambda} \leq 0.035) \).

- If \( K_c \gg 1 \): separation important, viscous forces can not be neglected. Further on if
  \[ \frac{\ell}{\lambda} \leq \frac{0.07}{A/\ell} \] so
  \[ \frac{\ell}{\lambda} \ll 1 \] ignore diffraction, i.e., the Froude-Krylov approximation is valid.

\[ F = \frac{1}{2} \rho \ell^2 U(t) |U(t)| C_D(R_e) \]

- Intermediate \( K_c \) - both viscous and inertial effects important, use Morrison’s formula.

\[ F = \frac{1}{2} \rho \ell^2 U(t)|U(t)| C_D(R_e) + \rho \ell^3 \dot{U} C_m(R_e, K_c) \]
I. Use: $C_D$ and $F - K$ approximation.

II. Use: $C_F$ and $F - K$ approximation.

III. $C_D$ is not important and $F - K$ approximation is not valid.