Roll Motion

Coupled with sway & yaw.

Linear damping due to radiated waves is small so Roll Motion can be LARGE.

- perfect circular shape with motion about its center
  - No waves are created

- Non-circular shape
  - Wave making is relatively small

Where does additional damping come from?

- FRICTION
  - Eddy-generation

- Surface friction \( \Rightarrow \) VISCOUS

- Eddy-generation

- Separation on appendages such as fins or keels
  - Also Eddy-generation

FRICTION

\[ F_t = C_t \frac{1}{2} \rho U^2 S \text{ surface area} \]

\[ F_t \propto U^2 \text{ - non-linear} \]
Roll Force \to F_q = \phi \frac{\dot{x}_4}{|\dot{x}_4|} \quad \phi = \text{constant}

\[ x_4 = \phi_0 \cos \omega t \quad \phi_0 = \text{roll amplitude} \]

Can we find an **Equivalent Linearization** of Roll Force?

\[ F_q^L = 2 \dot{x}_4 \]

Where coefficient \( \epsilon \) is determined such that the energy per cycle spent by \( F_q \) is the same as from the linearized \( F_q^L \to \)

\[ \int_0^T F_q \dot{x}_4 \, dt = \int_0^T F_q^L \dot{x}_4 \, dt \]

\[ \int_{\text{integral}} \text{over one period (cycle)} \]

\[ x_4 = \phi_0 \omega \sin \omega t \]

\[ \phi \omega^5 \phi_0^3 \int_0^{2\pi} \sin^2 \theta |\sin \theta| \, d\theta = \epsilon \omega^3 \phi^2_0 \int_0^{2\pi} \sin^2 \theta \, d\theta \]

\[ \int_0^{2\pi} \sin^2 \theta |\sin \theta| \, d\theta = 2 \int_0^{\pi} \sin^3 \theta \, d\theta = \frac{4}{3} \]

\[ \int_0^{2\pi} \sin^2 \theta \, d\theta = \frac{2\pi}{2} \]