Problem 1:

\[ f(t) = \frac{1}{2} \left( \frac{\partial}{\partial t} A U(t) / \text{ult} \right) t + (m + ma) \Delta t \]

Morrison's equation is not linear in general.

Choverer, thinking back to 13.021, there are instances when the inertial force dominates, and thus the non-linear drag component can be neglected.

Problem 2:

Impulse response \( h(t) = \delta(t - t_0) \)

\[ \int_{-\infty}^{\infty} h(t - t_0) dt = 1 \text{ delta func.} \]

\[ H(w) = \text{Fourier Transform} (h(t)) \]

\[ = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt \]

\[ = hi \int_{-\infty}^{\infty} u(t - t_0) e^{-i\omega t} dt \]

\[ \uparrow \text{constant} \]

Recall property of delta func.

\[ \int_{-\infty}^{\infty} u(t - t_0) f(t) dt = f(t_0) \]
Therefore $H(\omega) = h_0 e^{-j\omega t}$

b) input $x(t) = x_0 \sin(\omega_0 t + \psi_0)$
output $y(t) = h(t) * x(t)$

- or -

$$y(t) = \frac{x_0}{|H(\omega_0)|} \sin \left( \omega_0 t + \psi_0 + \angle H(\omega_0) \right)$$

$$H(\omega_0) = h_0 e^{-j\omega_0 t} = |H(\omega_0)| e^{j\angle H(\omega_0)}$$

$$y(t) = x_0 h_0 \sin \left( \omega_0 t + \psi_0 + (-\omega_0 t) \right)$$

$$\Rightarrow y(t) = x_0 h_0 \sin(\omega_0 (t-t_0) + \psi_0)$$

Problem 3:

- Circular orbits
- Exponential decay in radius to $r = -\infty$
3b) pressure under waves

Dynamic pressure $= \rho \frac{\partial \Phi}{\partial t}$

Hydrostatic $p = \rho g z$

$$\rho(z, t) = \rho_0 e^{-k z} \left[ \eta(z=0, t) + \rho g z \right]$$

where $\eta = a \cos(\beta x - \omega t)$

$$\eta(0, t) = a \cos(\omega t) \quad [\cos(-a) = \cos(+a)]$$

pressure is isotropic and acts everywhere normal to surface.

3c) Surge motion $x_1$

$$x_1 = x_{10} \cos(\omega t)$$

Equation of motion

$$(m + M_a) \ddot{x}_1 + B_n \dot{x}_1 + C_n x_1 = f(t)$$

mass linear tension/length of thread

3d) $H(\omega) = -\frac{(m + M_a) \omega^2 + i \omega B_n + C_n}{\omega^2} \frac{1}{2 \pi}$$
Problem 4

a) historical wave data
   - Ideally wave measurements over span of at least several decades
   - Info on wind patterns
   - Geological info about coastal shorelines & underwater formations that may affect wave development or propagation
   - From this we can get significant wave height etc...

b) Wind limits phase speed

   \[ C_w = \frac{U_w}{K} = \frac{g}{\omega} = 4 \text{ m/s} \]

   \[ \text{deepwater: } \omega^2 = gK \Rightarrow \omega = \frac{g}{K} \]

   \[ \omega = \frac{g}{U_w} = \frac{10}{4} = 2.5 \text{ rad/s} \]

c) \[ M_o = 12 \cdot (0.5) + 24 \cdot (0.5) + 18 \cdot (0.5) + 6 \cdot (0.5) \]

   \[ S(\omega) = 2.5 \text{ rad/s} \]

   \[ M_o = 30 \text{ m}^2 \text{ (rad/s)} \]

   \[ M_2 = \sum_i \omega_i^2 S_i(\omega) d\omega_i = 45.75 \text{ m}^2 \text{ (rad/s)} \]

   \[ M_4 = \sum_i \omega_i^4 S_i(\omega) d\omega_i = 105.95 \text{ m}^2 \text{ (rad/s)}^4 \]

   \[ \varepsilon = 0.58 \]

   \[ \text{Significant wave height } \xi = 4 \sqrt{M_o} = 21.9 \text{ m} \]

   V. large admitted.


d) \( \eta = \frac{3}{\text{day}} = \frac{3}{24 \times 60 \times 60} \text{ hrs, min, sec} \) = \( \frac{3}{86400} \text{ sec} \)

\( \eta = 3.47 \times 10^{-5} \text{ m}^2/\text{m} \)

\( \eta = \frac{1}{2\pi} \sqrt{\frac{M_0}{H_0}} e^{-\frac{L_0^2}{2H_0}} \)

\( \quad = \frac{1}{2\pi} \sqrt{\frac{45.75}{30}} \quad e^{-\frac{L_0^2}{2 \times 30}} = 3.47 \times 10^{-5} \)

Solve for \( L_0 \to \)

\[ e^{-\frac{L_0^2}{160}} = \frac{3.47 \times 10^{-5}}{0.1965} \]

\[ \ln \left( e^{-\frac{L_0^2}{160}} = 0.001766 \right) \]

\[ -\frac{L_0^2}{160} = -6.34 \]

\[ L_0^2 = 380.35 \text{ m}^2 \]

\[ \therefore L_0 = 19.5 \text{ m} \]

e) well first off \( S \) is quite large, probably not realistic...

Secondly you want windmill high enough to be out of mind. Boundary layer above...

Ocean waves (regardless of 4m getting wet) (not 4m, mind assuming a 1km fetch 5 \( \times \) 5m) to aside...

for smooth surface turbulent flow