1. Determine whether the following systems are linear and/or time invariant.
   a. \[ y(t) = \int_{0}^{t+\alpha} u(s) ds \]
   b. \[ y(t) = \int_{t-\alpha}^{t+\alpha} [u(s)]^2 ds \]
   c. \[ y(t) = \alpha \frac{du(t)}{dt} \left| \frac{du(t)}{dt} \right| \]
   d. \[ \alpha \ddot{y}(t) + \beta \dot{y}(t) + \gamma y(t) = u(t) \]

2. Determine whether the following systems are LTI systems.
   a. \[ a_1 \cos \omega t \rightarrow \rightarrow a_2 \cos(\omega t + \phi) \]
   b. \[ \sin 5t \rightarrow \rightarrow 2 \cos(10t + \pi) \]

3. Fourier Transform
   a. Find the Fourier Transform of \( f(t) = u_o(t - \tau) \).
   b. Given that \( f(x) \rightarrow 0 \) as \( |x| \rightarrow \infty \), and the Fourier Transform of \( f(x) \) is \( \tilde{f}(\alpha) \), what is the Fourier Transform of \( \frac{df}{dx} \)? (Hint: Use partial integration.)
   c. Given that \( \frac{df}{dx} \rightarrow 0 \) as \( |x| \rightarrow \infty \), what is the Fourier Transform of \( \frac{d^2 f}{dx^2} \)?
4. Transfer Function:
   a. Given the following linear system:

   \[ m \ddot{x} + c \dot{x} + kx = f(t) \]

   where input \( f(t) = \text{Re}\{F e^{j\omega t}\} \) and response \( x(t) = \text{Re}\{X e^{j\omega t}\} \), and \( X \) and \( F \) are both complex quantities, find the transfer function \( H(\omega) \).

   b. Using the same system, for which you have just found the transfer function, if the input is \( \alpha f_1(t) + \beta f_2(t) \) determine the system output, \( x(t) \).

5. Convolution

Perform the following convolutions (from page 2.11 in Triantafyllou and Chryssostomidis, *Environmental Description, Force Prediction and Statistics for Design Applications in Ocean Engineering*):

6. A linear time-invariant system has a transfer function \( H(\omega) \), show that when the input is sinusoidal with frequency \( \omega_o \), i.e.

   \[ f(t) = f_o \cos(\omega_o t + \psi) \]

   the output is also a sinusoidal function with the same frequency.