1) Perform a numerical integration on the following function

\[ I = \int_{0}^{5} \frac{\cos^2(x) + 3}{e^x + x^3} \, dx \quad \text{From } x=0 \text{ to } x=5 \]

Using:

a) Trapezoidal rule with 100 steps

b) Trapezoidal rule with 200 steps

c) Above results + Richardson extrapolation equation
   (This is a 2 iteration Romberg integration)

d) Using a 2 point Gaussian Quadrature method

2) Implement an Euler method solution in Matlab code to solve the differential equation:

\[ \frac{di}{dt} = 1.5 + 2.5i \]

a) Compute and plot the exact solution of the integral from time t=0 to t=3.

b) Implement a central difference Euler solution ti I as a function of time with initial conditions i=0 at t=0 up to a time of t=3.

c) Run the euler solution for various stepsizes from 0.2 to 0.002

d) Produce a plot showing the exact solution and a few different stepsizes showing convergence with stepsize.

3) A propeller on a ship body has a diameter of 3 meters and has an engine capable of producing 10,000 kW.

a) Plot the limiting top speed for this ship as a function of drag on the ship. (assume a lower limit on drag of 200kN)

b) If the ship moves at 12 knots what is the maximum drag that this hull could have?

c) At this speed estimate the wake diameter exiting the propeller far downstream?
4) Fun with Functions: Write matlab functions which perform the following computations: Pass in your m-files as well as test case result.

   a) Compute the unit normal vector given 3 points in space:
      \[ P_1 = [X_1, Y_1, Z_1] \quad P_2 = [X_2, Y_2, Z_2] \quad \text{and} \quad P_3 = [X_3, Y_3, Z_3] \]
      Test the function by computing the normal vector on a plane of known orientation.

   b) Compute the velocity vector induced by a vortex segment of input strength \( \Gamma \) at a point in space given the end points of the vortex:
      \[ PV_1 = (X_{V_1}, Y_{V_1}, Z_{V_1}) \quad \text{and} \quad PV_2 = (X_{V_2}, Y_{V_2}, Z_{V_2}) \]
      and the point in space \( P = [X, Y, Z] \). Test the function with a long unit strength vortex on the z axis and a point P unit distance along the x-axis. The result should be close to the 2D vortex velocity result.