1. (20%) A two dimensional plane progressive wave with amplitude $A$ and wavelength $\lambda$ interacts with a neutrally buoyant submerged circle of radius $a$, as shown in the figure.

   \[ A, \lambda \]

   \[ a \]

   \[ \lambda \gg a \]

   a) Assuming that the wavelength $\lambda$ is large compared to the circle radius and the heave and surge wave damping coefficients of the circle are zero, show that the circle heave and surge motions are identical to those of a fluid particle with the circle absent.

   b) Assume now that the circle heave and surge wave damping coefficient is $B > 0$ and independent of the frequency. Derive the modulus and phase of the heave and surge motions of the circle in regular waves with $\lambda \gg a$.

2. (20%) A pressure gauge is placed on the bottom of a channel of uniform depth $H$, in lieu of a "police radar", in order to detect the speed of ships cruising on the surface. Assume for the purpose of this problem that the flow is two-dimensional.

   When a ship passes over, the instrument detects a pressure signal which has the shape shown in the figure.

   \[ P(t) \]

   \[ T \]

   \[ t \]

   a) Derive a relation between the ship speed $U$ and the period $T$ of the pressure signal.

   b) The amplitude of the pressure signal effectively disappears when the period $T$ drops below a critical period $T^*$. Derive a relation between $T^*$ and the water depth $H$.

   c) Derive a new value for $T$ when a current with velocity $u$ flows against the ship?
3. (30%) A ship with length L and effectively uniform cross section along its length is cruising at speed U in head waves.

a) Derive an expression for the wavelength at which the modulus of the pitch exciting moment attains its maximum value.

b) Derive an expression for the wavelength at which the pitch moment modulus is effectively zero.

c) Generalize your results in questions a) and b) above when the same ship is cruising in oblique waves with heading $\beta$.

d) How do your answers in questions a-c depend upon the ship speed?

4. (30%) An underwater offshore storage tank shaped like a vertical circular cylinder with radius $a$ and height $T$ is sitting on the ocean floor with depth $H$, as shown in the figure. Plane progressive waves of amplitude $A$ and wavelength $\lambda$ are incident upon the cylinder.

![Diagram of a cylindrical tank with incident waves and a point B located on the weather side of the tank.]

a) Derive an expression for the overturning pitch moment amplitude and phase exerted by the ambient waves upon the storage tank. Derive the moment about an origin located at point A.

b) In order to carry out the tank structural design we need estimates of the linear hydrodynamic pressure over its surface. Derive an expression of the total linear pressure amplitude and phase at the point B located on the weather side of the tank.

c) Assuming that the wavelength $\lambda$ is large compared to the cylinder radius $a$, estimate the mean surge drift force exerted on the cylinder by the ambient waves.