Problem 5.29

This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin

The sketch shows a lawn sprinkler with two horizontal arms of radial length \( R \), at the termination of which are nozzles (exit Area \( A_2 \)) pointing in a direction which is at an angle \( \theta \) relative to the tangent of a circumferential line, as shown. The sprinkler is free to rotate, but the bearing on which it is mounted exerts a torque \( k \omega \) in the direction opposing the rotation, \( \omega \) being the angular rate of rotation. A constant volume flow rate \( Q \) passes through the sprinkler, the flow being incompressible at density \( \rho \).

- (a) Find an expression for the steady-state angular velocity \( \omega \) of sprinkler in terms of the given quantities \( R, A_2, \theta, Q, \rho, \) and \( k \).

- (b) In the steady state, what is the velocity vector of the fluid emerging from the nozzles, as seen by an observer in the non-rotating reference frame? What is the fluid velocity at the nozzle vent if the bearing is frictionless (\( k = 0 \))? 

- (c) If the pipe area at station 1 near the bearing is \( A_1 \), and the flow from that point to the nozzles is inviscid, what gage pressure is required at station 1 to sustain the flow rate in this steady state?