Problem 8.13

This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin

Consider a gas bubble of fixed mass and radius $R(t)$ which is expanding or contracting in an infinite sea of incompressible liquid. The speed of the interface is $dR/dt$. The local Eulerian coordinate in the liquid is $r$. Let $p_R$, $p$, and $p_\infty$ be, respectively the pressure at $r = R$ (on the liquid side of the interface), at $r = r$, and at $r = \infty$.

(a) Determine the viscous contribution to the normal stress $\tau_{rr}$ in the liquid.

(b) Show that the dimensionless overpressure, $(p_R - p_\infty)/\rho(dR/dt)^2$, is independent of whether the fluid is viscous or inviscid.