Consider a furnace of height $H$ with a tall cylindrical smoke stack of diameter $d$ ($d \ll H$) and height $h$ ($h \gg H$). Air, an ideal gas ($P = \rho RT$), enters the furnace at atmospheric density and temperature and at local atmospheric pressure. Between stations 1 and 2, heat is added at constant pressure and the air temperature is raised by an amount $\Delta T$. Thereafter, heat addition is negligible and the air rises through the stack at a sensibly constant density.

(a) On the assumption that viscous effects are negligible, derive an expression for the steady mass flow rate of air drawn by a stack of given height, $h$, in terms of the temperature rise in the furnace.

(b) If the chimney were capped off at the top, what would be the pressure differential across the cap, assuming that $\Delta T$ would not be altered by the flow stoppage?

Note: The height $h$ of the stack is small compared with the length $RT_a/g$ over which the atmosphere density falls by $1/e$ (see Problem 1.8). Hence, gravitational density changes can be neglected.