Problem 2.07
This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin

A drop of liquid of volume $V$ is squeezed between two parallel smooth plates until the liquid thickness $h$ is very small compared with the liquid’s radial extent $R$. The liquid/plate/air contact angle $\alpha$, and the liquid/air surface tension is $\sigma$. Gravitational effects are negligible.

(a) Derive an expression for the downward force $F$ required to hold the plates in position. Express $F$ in terms $V$, $\alpha$, $\sigma$, and $R$.

(b) If $\alpha = \pi$ radians (a perfectly nonwetting situation) and $T = 0.07 \text{N/m}$, say (representing a clean air-water interface), what downward force is required to press a 3 mm$^3$ drop of liquid into a thin disc or radius $R = 2 \text{cm}$?