2.29 Numerical Fluid Mechanics
Spring 2015 – Lecture 19

REVIEW Lecture 18:

• Solution of the Navier-Stokes Equations
  – Discretization of the convective and viscous terms
  – Discretization of the pressure term
  – Conservation principles
    • Momentum and Mass
    • Energy
      \[
      \frac{\partial \rho \vec{v}}{\partial t} + \nabla (\rho \vec{v} \cdot \vec{v}) = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}
      \nabla \cdot \vec{v} = 0
      \]
  \[
  \tilde{p} = p - \rho g \cdot \vec{r} + \mu \frac{2}{3} \nabla \cdot \vec{u} \quad (p \tilde{e}_i - \rho g_i \tilde{e}_i + \frac{2}{3} \mu \frac{\partial u_j}{\partial x_j} \tilde{e}_i)
  \]
  \[
  \int_S -\tilde{p} \tilde{e}_i \tilde{n} dS
  \]

  – Choice of Variable Arrangement on the Grid
    • Collocated and Staggered

  – Calculation of the Pressure
    \[
    \nabla \cdot \nabla p = \nabla^2 p = -\nabla \cdot \left( \frac{\partial \rho \vec{v}}{\partial t} - \nabla \cdot \left( \nabla (\rho \vec{v} \cdot \vec{v}) \right) + \nabla \cdot \left( \mu \nabla^2 \vec{v} \right) + \nabla \cdot (\rho \vec{g}) \right) = -\nabla \cdot (\nabla (\rho \vec{v} \cdot \vec{v}))
    \]
    \[
    \Rightarrow \frac{\partial \rho \vec{v}}{\partial t} = -\frac{\partial}{\partial x_i} \left( \frac{\partial p}{\partial x_i} \right) = -\frac{\partial}{\partial x_i} \left( \frac{\partial (\rho u_i u_j)}{\partial x_j} \right)
    \]
REVIEW Lecture 18, Cont’d:

• Solution of the Navier-Stokes Equations
  – Pressure Correction Methods:
    • i) Solve momentum for a known pressure leading to new velocity, then
    • ii) Solve Poisson to obtain a corrected pressure and
    • iii) Correct velocity (and possibly pressure), go to i) for next time-step.

• A Forward-Euler Explicit (Poisson for $p$ at $t_n$, then mom. for velocity at $t_{n+1}$)
• A Backward-Euler Implicit

\[
(\rho u_i)^{n+1} - (\rho u_i)^n = \Delta t \left( -\frac{\delta (\rho u_i u_j)}{\delta x_j} + \frac{\delta \tau_{ij}}{\delta x_j} - \frac{\delta p^{n+1}}{\delta x_i} \right) \quad \frac{\delta}{\delta x_i} \left( \frac{\delta p^{n+1}}{\delta x_i} \right) = \frac{\delta}{\delta x_i} \left( -\frac{\delta (\rho u_i u_j)}{\delta x_j} + \frac{\delta \tau_{ij}}{\delta x_j} \right)
\]

– Nonlinear solvers, Linearized solvers and ADI solvers

• Steady state solvers, implicit pressure correction schemes: iterate using
  – Outer iterations:
    \[
    A^{u^{m}} u_{i}^{m} = b_{u_{i}}^{m-1} - \frac{\delta p^{m-1}}{\delta x_i} \quad \text{but require} \quad A^{u^{m}} u_{i}^{m} = b_{u_{i}}^{m} - \frac{\delta p^{m}}{\delta x_i} \quad \frac{\delta u_{i}^{m}}{\delta x_i} = 0 \implies 0 \approx \frac{\delta \tilde{u}_{i}^{m}}{\delta x_i} - \frac{\delta}{\delta x_i} \left( (A^{u^{m}})^{-1} \frac{\delta p^{m}}{\delta x_i} \right)
    \]
  – Inner iterations:
    \[
    A^{u^{m}} u_{i}^{m} = b_{u_{i}}^{m} - \frac{\delta p^{m}}{\delta x_i}
    \]
REVIEW Lecture 18, Cont’d:

• Solution of the Navier-Stokes Equations
  – Projection Correction Methods:
    – Construct predictor velocity field that does not satisfy continuity, then correct it using a pressure gradient
    – Divergence producing part of the predictor velocity is “projected out”

• Non-Incremental:
  – No pressure term used in predictor momentum eq.

• Incremental:
  – Old pressure term used in predictor momentum eq.

• Rotational Incremental:
  – Old pressure term used in predictor momentum eq.
  – Pressure update has a rotational correction: \( p^{n+1} = p^n + p' = p^n + \delta p^{n+1} + f(u') \)
TODAY (Lecture 19)

• Solution of the Navier-Stokes Equations
  – Pressure Correction Methods
    • Projection Methods
      – Non-Incremental, Incremental and Rotational-incremental Schemes
  – Fractional Step Methods:
    • Example using Crank-Nicholson
  – Streamfunction-Vorticity Methods: scheme and boundary conditions
  – Artificial Compressibility Methods: scheme definitions and example
  – Boundary Conditions: Wall/Symmetry and Open boundary conditions

• Time-Time-Marching Methods and ODEs. – Initial Value Problems
  – Euler’s method
  – Taylor Series Methods
    • Error analysis
  – Simple 2nd order methods
    • Heun’s Predictor-Corrector and Midpoint Method (belong to Runge-Kutta’s methods)
References and Reading Assignments


Rotational Incremental (Timmermans et al, 1996):

- Old pressure term used in predictor momentum equation
- Correct pressure based on continuity: \( p^{n+1} = p^n + p' = p^n + \delta p^{n+1} + f(u') \)
- Update velocity using pressure increment in momentum equation

\[
\left( \rho u_i^* \right)^{n+1} = \left( \rho u_i \right)^n + \Delta t \left( -\frac{\delta (\rho u_i u_j)^{n+1}}{\delta x_j} + \frac{\delta \tau_{ij}^{n+1}}{\delta x_j} - \frac{\delta p^n}{\delta x_i} \right); \left. \left( \rho u_i^* \right)^{n+1} \right|_{\partial D} = (bc)
\]

\[
\left( \rho u_i \right)^{n+1} = \left( \rho u_i^* \right)^{n+1} - \Delta t \frac{\delta (\delta p^{n+1})}{\delta x_i} = 0
\]

\[
\left( \rho u_i \right)^{n+1} = \left( \rho u_i^* \right)^{n+1} - \Delta t \frac{\delta (\delta p^{n+1})}{\delta x_i} = 0
\]

\[
p^{n+1} = p^n + \delta p^{n+1} - \mu \frac{\delta}{\delta x_i} \left( \left( u_i^* \right)^{n+1} \right)
\]

Notes:
- this scheme accounts for \( u' \) in the pressure eqn.
- It can be made into a SIMPLE-like method, if iterations are added
- Again, the advection term can be explicit or implicit. The rotational correction to the left assumes explicit advection
Other Methods: Fractional Step Methods

• In the previous methods, pressure is used to:
  – Enforce continuity: it is more a mathematical variable than a physical one
  – Fill the RHS of the momentum eqns. explicitly (predictor step for velocity)

• The fractional step methods (Kim and Moin, 1985) generalize ADI
  – But works on term-by-term (instead of dimension-by-dimension). Hence, does not necessarily use pressure in the predictor step

  – Let’s write the NS equations a in symbolic form:

\[
u_{i}^{n+1} = u_{i}^{n} + (C_{i} + D_{i} + P_{i}) \Delta t
\]

where \(C_{i}, D_{i}\) and \(P_{i}\) represent the convective, diffusive and pressure terms

  – The equation is readily split into a three-steps method:

\[
\begin{align*}
    u_{i}^{*} &= u_{i}^{n} + C_{i} \Delta t \\
    u_{i}^{**} &= u_{i}^{*} + D_{i} \Delta t \\
    u_{i}^{n+1} &= u_{i}^{**} + P_{i} \Delta t
\end{align*}
\]

  – In the 3rd step, the pressure gradient ensures \(u_{i}^{n+1}\) satisfy the continuity eq.
Fractional Step Methods, Cont’d

• Many variations of Fractional step methods exists
  – Pressure can be a pseudo-pressure (depends on the specific steps, i.e. what is in $u^*_i, P_i$)
  – Terms can be split further (one coordinate at a time, etc.)
  – For the time-marching, Runge-Kutta explicit, direct 2nd order implicit or Crank-Nicholson scheme are often used
  – Linearization and ADI are also used
  – Used by Choi and Moin (1994) with central difference in space for direct simulations of turbulence (Direct Navier Stokes, DNS)

• Next, we describe a scheme similar to that of Choi and Moin, but using Crank-Nicholson
Fractional Step Methods: Example based on Crank-Nicholson

• In the first step, velocity is advanced using:
  
  \[(\rho u_i)^* - (\rho u_i)^n = \Delta t \left( \frac{H(u_i^n) + H(u_i^*)}{2} - \frac{\delta p^n}{\delta x_i} \right)\]

  – Pressure from the previous time-step
  – Convective, viscous and body forces are represented as an average of old and new values (Crank-Nicolson)
  – Nonlinear equations ⇒ iterate, e.g. Newton’s scheme used by Choi et al (1994)

• Second-step: Half the pressure gradient term is removed from \(u_i^*\), to lead \(u_i^{**}\)

  \[(\rho u_i)^{**} - (\rho u_i)^* = -\Delta t \left( -\frac{1}{2} \frac{\delta p^n}{\delta x_i} \right)\]

• Final step: use half of the gradient of the still unknown new pressure

  \[(\rho u_i)^{n+1} - (\rho u_i)^{**} = -\Delta t \left( \frac{1}{2} \frac{\delta p^{n+1}}{\delta x_i} \right)\]

• New velocity must satisfy the continuity equation (is divergence free):

  – Taking the divergence of final step:

    \[\frac{\delta}{\delta x_i} \left( \frac{\delta p^{n+1}}{\delta x_i} \right) = 2 \frac{\delta (\rho u_i)^{**}}{\Delta t \delta x_i}\]

  – Once \(p^{n+1}\) is solved for, the final step above gives the new velocities
Fractional Step Methods:  
Example based on Crank-Nicholson

• Putting all steps together:

\[(\rho u_i)^{n+1} - (\rho u_i)^n = \Delta t \left[ \frac{H(u_i^n) + H(u_i^*)}{2} - \frac{1}{2} \left( \frac{\delta p^n}{\delta x_i} + \frac{\delta p^{n+1}}{\delta x_i} \right) \right] \]

– To represent Crank-Nicolson correctly, \(H(u_i^*)\) should be \(H(u_i^{n+1})\)

– However, we can show that the splitting error, \(u_i^{n+1} - u_i^*\), is 2\(^{nd}\) order in time and thus consistent with C-N’s truncation error: indeed, subtract the first step from the complete scheme, to obtain,

\[(\rho u_i)^{n+1} - (\rho u_i)^* = -\Delta t \left( \frac{\delta p^{n+1}}{\delta x_i} - \frac{\delta p^n}{\delta x_i} \right) \approx -\Delta t^2 \frac{\delta}{\delta x_i} \left( \frac{\delta p}{\delta t} \right) \]

– With this, one also obtains:

\[(\rho u_i)^{n+1} - (\rho u_i)^* = -\frac{\Delta t}{2} \frac{\delta (p^{n+1} - p^n)}{\delta x_i} = -\frac{\Delta t}{2} \frac{\delta (p')}{\delta x_i} \]

which is similar to the final step, but has the form of a pressure-correction on \(u_i^*\). This later eq. can be used to obtain a Poisson eq. for \(p'\) and replace that for \(p^{n+1}\)

• Fractional steps methods have become rather popular

  – Many variations, but all are based on the same principles (illustrated by C-N here)

  – Main difference with SIMPLE-type time-marching schemes: SIMPLE schemes solve the nonlinear pressure and momentum equations several times per time-step in outer iterations (iterative nonlinear solve)
Incompressible Fluid

Vorticity Equation

\( \vec{\omega} \equiv \text{curl} \vec{V} \equiv \nabla \times \vec{V} \)

Navier-Stokes Equation

\[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \vec{V} \]

\textbf{curl} of Navier-Stokes Equation

\[ \frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla) \vec{V} + \nu \nabla^2 \vec{\omega} \]
Streamfunction-Vorticity Methods

- For incompressible, 2D flows with constant fluid properties, NS can be simplified by introducing the streamfunction $\psi$ and vorticity $\omega$ as dependent variables
  
  \[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad \text{and} \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (\omega = \nabla \times \mathbf{v}) \]

  - Streamlines (lines tangent to velocity): constant $\psi$
  - Vorticity vector is orthogonal to plane of the 2D flow
  - 2D continuity is automatically satisfied: \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \)

- In 2D, substituting $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ in $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ leads to the kinematic condition:
  
  \[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \]

- The vertical component of the vorticity dynamical equation leads:
  
  \[ \rho \frac{\partial \omega}{\partial t} + \rho u \frac{\partial \omega}{\partial x} + \rho v \frac{\partial \omega}{\partial y} = \mu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \]
Streamfunction-Vorticity Methods, Cont’d

• Main advantages:
  – Pressure does not appear in either of these equations!
  – 2D-NS has been replaced by a set of 2 coupled PDEs
    • Instead of 2 velocities and 1 pressure, we have only two dependent variables

• Explicit solution scheme
  – Given initial velocity field, compute vorticity by differentiation
  – Use this vorticity $\omega^n$ in the RHS of the dynamical equation for vorticity, to obtain $\omega^{n+1}$
    – With $\omega^{n+1}$ the streamfunction $\psi^{n+1}$ can be obtained from the Poisson equation
      • With $\psi^{n+1}$, we can differentiate to obtain the velocity
  – Continue to time $n+2$, and so on

• One issue with this scheme: boundary conditions
Streamfunction-Vorticity Methods, Cont’d

Boundary conditions

- Boundary conditions for $\psi$
  - Solid boundaries are streamlines and require: $\psi = \text{constant}$
  - However, values of $\psi$ at these boundaries can be computed only if velocity field is known

- Boundary conditions for $\omega$
  - Neither vorticity nor its derivatives at the boundaries are known in advance
  - For example, at the wall: \( \omega_{\text{wall}} = -\tau_{\text{wall}} / \mu \) since \( \tau_{\text{wall}} = \mu \frac{\partial u}{\partial y}_{\text{wall}} \)
    - Vorticity at the wall is proportional to the shear stress, but the shear stress is often what one is trying to compute
  - Boundary values for $\omega$ can be obtained from \( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \)
    - i.e. one-sided differences at the wall: \( \frac{\partial^2 \psi}{\partial n^2} = -\omega \)
    - but this usually converges slowly and can require refinement
  - Discontinuities also occur at corners
– Discontinuities also occur at corners for vorticity
  • The derivatives \( \frac{\partial v}{\partial x} \) and \( \frac{\partial u}{\partial y} \)
    are not continuous at A and B
  • This means special treatment for
    \[
    \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
    \]
    e.g. refine the grid at corners

• Vorticity-streamfunction approach useful in 2D, but is now less popular because extension to 3D difficult
  – In 3D, vorticity has 3 components, hence problem becomes as/more expensive as NS
  – Streamfunction is still used in quasi-2D problems
    • for example, in the ocean or in the atmosphere, but even there, it has been replaced by level-based models with a free-surface (no steady 2D continuity)
Artificial Compressibility Methods

• Compressible flow is of great importance (e.g. aerodynamics and turbine engine design)
• Many methods have been developed (e.g. MacCormack, Beam-Warming, etc)
• Can they be used for incompressible flows?
• Main difference between incompressible and compressible NS is the mathematical character of the equations
  – Incompressible eqs.: no time derivative in the continuity eqn: \( \nabla \cdot \mathbf{v} = 0 \)
    • They have a mixed parabolic-elliptic character in time-space
  – Compressible eqs.: there is a time-derivative in the continuity equation:
    • They have a direct hyperbolic character:
      \[
      \frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0
      \]
    • Allow pressure/sound waves
  – How to use methods for compressible flows in incompressible flows?
Artificial Compressibility Methods, Cont’d

• Most straightforward: Append a time derivative to the continuity equation
  – Since density is constant, adding a time-rate-of-change for \( \rho \) not possible
  – Use pressure instead (linked to \( \rho \) via an eqn. of state in the general case):
    \[
    \frac{1}{\beta} \frac{\partial p}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0
    \]
    where \( \beta \) is an artificial compressibility parameter (dimension of velocity\(^2\))

• Its value is key to the performance of such methods:
  – The larger/smaller \( \beta \) is, the more/less incompressible the scheme is
  – Large \( \beta \) makes the equation stiff (not well conditioned for time-integration)

• Methods most useful for solving steady flow problem (at convergence: \( \frac{\partial p}{\partial t} = 0 \))
or inner-iterations in dual-time schemes.
  – To solve this new problem, many methods can be used, especially
    • Time-marching schemes: what we have seen & will see (R-K, multi-steps, etc)
    • Finite differences or finite volumes in space
    • Alternating direction method is attractive: one spatial direction at a time
Artificial Compressibility Methods, Cont’d

• Connecting these methods with the previous ones:
  – Consider the intermediate velocity field \((\rho u_i^*)^{n+1}\) obtained from solving momentum with the old pressure
  – It does not satisfy the incompressible continuity equation:
    \[ \frac{\delta (\rho u_i^*)^{n+1}}{\delta x_i} = \frac{\partial \rho^*}{\partial t} \]
    • There remains an erroneous time rate of change of mass flux
      \[ \Rightarrow \text{method needs to correct for it} \]
• Example of an artificial compressibility scheme
  – Instead of explicit in time, let’s use implicit Euler (larger time steps for stiff term with large \(\beta\))
    \[ \frac{p^{n+1} - p^n}{\beta \Delta t} + \left[ \frac{\delta (\rho u_i)}{\delta x_i} \right]^{n+1} = 0 \]
  – Issue: velocity field at \(n+1\) not known \(\Rightarrow\) coupled \(u_i\) and \(p\) system solve
  – To decouple the system, one could linearize about the old (intermediate) state and transform the above equation into a Poisson equation for the pressure or pressure correction!
Artificial Compressibility Methods:
Example Scheme, Cont’d

- **Idea 1**: expand unknown $u_i$ using Taylor series in pressure derivatives

\[(\rho u_i)^{n+1} \approx (\rho u_i^*)^{n+1} + \left[ \frac{\delta (\rho u_i^*)}{\delta p} \right]^{n+1} (p^{n+1} - p^n) \quad (p^{*n+1} = p^n)\]

- Inserting $(\rho u_i)^{n+1}$ in the continuity equation leads an equation for $p^{n+1}$

\[\frac{p^{n+1} - p^n}{\beta \Delta t} + \frac{\delta}{\delta x_i} \left[ (\rho u_i^*)^{n+1} + \left[ \frac{\delta (\rho u_i^*)}{\delta p} \right]^{n+1} (p^{n+1} - p^n) \right] = 0\]

- Expressing $\left[ \frac{\delta (\rho u_i^*)}{\delta p} \right]^{n+1}$ in terms of $\frac{\delta p^{n+1}}{\delta x_i}$ using N-S, this is a Poisson-like eq. for $p^{n+1} - p^n$!

- **Idea 2**: utilize directly

\[(\rho u_i)^{n+1} \approx (\rho u_i^*)^{n+1} + \left[ \frac{\delta (\rho u_i^*)}{\delta \left( \frac{\delta p}{\delta x_i} \right)} \right]^{n+1} \left( \frac{\delta p^{n+1}}{\delta x_i} - \frac{\delta p^n}{\delta x_i} \right)\]

- Then, still take divergence of $(\rho u_i^*)^{n+1}$ and derive Poisson-like equation

- **Ideal value of $\beta$ is problem dependent**

  - The larger the $\beta$, the more incompressible. Lowest values of $\beta$ can be computed by requiring that pressure waves propagate much faster than the flow velocity or vorticity speeds
Numerical Boundary Conditions for N-S eqns.: Velocity

- At a wall, the no-slip boundary condition applies:
  - Velocity at the wall is the wall velocity (Dirichlet)
  - In some cases (e.g., fully-developed flow), the tangential velocity is constant along the wall. By continuity, this implies no normal viscous stress:

\[
\frac{\partial u}{\partial x} \bigg|_{\text{wall}} = 0 \quad \Rightarrow \quad \frac{\partial v}{\partial y} \bigg|_{\text{wall}} = 0
\]

\[\Rightarrow \quad \tau_{yy} = 2\mu \frac{\partial v}{\partial y} \bigg|_{\text{wall}} = 0\]

- For the shear stress:

\[
F_s^{\text{shear}} = \int_{S_s} \tau_{xy} \, dS = \int_{S_S} \mu \frac{\partial u}{\partial y} \, dS \approx \mu_S S_S \frac{u_P - u_S}{y_P - y_S}
\]

- At a symmetry plane, it is the opposite:

- Shear stress is null:

\[
\tau_{xy} = \mu \frac{\partial u}{\partial y} \bigg|_{\text{sym}} = 0 \quad \Rightarrow \quad F_s^{\text{shear}} = 0
\]

- Normal stress is non-zero:

\[
\tau_{yy} = 2\mu \frac{\partial v}{\partial y} \bigg|_{\text{sym}} \neq 0 \quad \Rightarrow \quad F_s^{\text{normal}} = \int_{S_s} \tau_{yy} \, dS = \int_{S_S} 2\mu \frac{\partial v}{\partial y} \, dS \approx 2\mu_S S_S \frac{v_P - v_S}{y_P - y_S}
\]
Numerical Boundary Conditions for N-S eqns.: Pressure

• Wall/Symmetry Pressure BCs for the Momentum equations
  – For the momentum equations with staggered grids, the pressure is not required at boundaries (pressure is computed in the interior in the middle of the CV or FD cell)
  – With collocated arrangements, values at the boundary for $p$ are needed. They can be extrapolated from the interior (may require grid refinement)

• Wall/Symmetry Pressure BCs for the Poisson equation
  – When the mass flux (velocity) is specified at a boundary, this means that:
    • Correction to the mass flux (velocity) at the boundary is also zero
    • This affects the continuity eq., hence the $p$ eq.: zero normal-velocity-correction $\Rightarrow$ often means gradient of the pressure-correction at the boundary is then also zero

(take the dot product of the velocity correction equation with the normal at the bnd)
Numerical BCs for N-S eqns: Outflow/Outlet Conditions

• Outlet often most problematic since information is advected from the interior to the (open) boundary

• If velocity is extrapolated to the far-away boundary, \( \frac{\partial u}{\partial n} = 0 \) e.g., \( u_E = u_p \),
  – It may need to be corrected so as to ensure that the mass flux is conserved (same as the flux at the inlet)
  – These corrected BC velocities are then kept fixed for the next iteration. This implies no corrections to the mass flux BC, thus a von Neumann condition for the pressure correction (note that \( p \) itself is linear along the flow if fully developed).
  – The new interior velocity is then extrapolated to the boundary, etc.
  – To avoid singularities for \( p \) (von Neumann at all boundaries for \( p \)), one needs to specify \( p \) at a one point to be fixed (or impose a fixed mean \( p \))

• If flow is not fully developed: \( \frac{\partial u}{\partial n} \neq 0 \) \( \Rightarrow \frac{\partial p'}{\partial n} \neq 0 \) \( \Rightarrow \) e.g. \( \frac{\partial^2 u}{\partial n^2} = 0 \) or \( \frac{\partial^2 p'}{\partial n^2} = 0 \)

• If the pressure difference between the inlet and outlet is specified, then the velocities at these boundaries can not be specified.
  – They have to be computed so that the pressure loss is the specified value
  – Can be done again by extrapolation of the boundary velocities from the interior: these extrapolated velocities can be corrected to keep a constant mass flux.

• Much research in OBC in ocean modeling
2.29 Numerical Fluid Mechanics
Spring 2015

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