SCATTERING FROM PARTICLES

\[ \overline{E}_2 = \overline{E}_i + \overline{E}_s \]
\[ \overline{H}_2 = \overline{H}_i + \overline{H}_s \]

\[ \langle S \rangle = \frac{1}{2} \text{Re} (\overline{E} \times \overline{H}^*) = \frac{1}{2} \text{Re} (\overline{E}_i \times \overline{H}_i^*) + \frac{1}{2} \text{Re} (\overline{E}_s \times \overline{H}_s^*) + \frac{1}{2} \text{Re} (\overline{E}_i \times \overline{H}_s^* + \overline{E}_s \times \overline{H}_i^*) \]

\[ \int \langle S \rangle \cdot d\overline{A} = \int \langle S_i \rangle \cdot d\overline{A} + \int \langle S_s \rangle \cdot d\overline{A} + \int \langle S_e \rangle \cdot d\overline{A} \]

\[ -W_a = 0 + W_s - W_e \]
\[ W = [\text{Power}] \]

\[ W_e = W_a + W_s \]

PHYSICAL PICTURE

DETECTOR WILL MEASURE LESS DUE TO INCOMING BEING SCATTERED AND ABSORBED.
CROSS-SECTIONS

SCATTERING CROSS-SECTION
\[ C_s = \frac{W_s}{I_i} \]

ABSORPTION
\[ C_a = \frac{W_a}{I_i} \]

EXTINCTION
\[ C_e = \frac{W_e}{I_i} \]

SCATTERING EFFICIENCY:
\[ Q_s = \frac{C_s}{A_e} \quad (A_e = \pi r^2) \]

ABSORPTION
\[ Q_a = \frac{C_a}{A_e} \]

EXTINCTION
\[ Q_e = \frac{C_e}{A_e} \]

ALBEDO:
\[ \omega_0 = \frac{Q_s}{Q_e} \]

PHASE FUNCTION \( (\phi) \)

\[ \phi(-\Omega' \rightarrow \Omega) = \frac{\text{POWER SCATTERED INTO } \Omega}{\text{POWER IN FROM SOLID } \Omega} \]

KNOSTROPIC CASE
ISOTROPIC SCATTERING

\[ \phi (\Omega' \to \Omega) = 1 \]

\[ \frac{1}{4\pi} \int \phi (\Omega' \to \Omega) \, d\Omega = 1 \]

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ALL RESULTS THUS FAR ARE FOR ANY GEOMETRY, NOT JUST SPHERES

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* SPHERICAL PARTICLES *

Mie Theory (1908, Gustav Mie)

\[ Q_a = \frac{Z}{X^2} \sum_{n=1}^{\infty} (2n+1) \text{Re} \left\{ a_n + b_n^2 \right\} \]

\[ Q_b = \frac{Z}{X^2} \sum_{n=1}^{\infty} (2n+1) \left( |a_n|^2 + |b_n|^2 \right) \]

\[ a_n = f_1 \left[ \psi_n, \xi_n, \mu, \nu \right] \]

\[ b_n = f_2 \left[ \right] \]
\[ \psi_n \] \text{ RICATTI-BESSSEL FUNCTIONS} \\

\[ \psi_{n+1}(x) = \frac{2n+1}{x} \]

\[ X \text{ = SIZE PARAMETER} \quad x = \frac{2\pi r}{\lambda_0} \quad \lambda_0 \text{ IS WAVELENGTH IN SURROUNDING MEDIA} \]

\[ m = \frac{N_1}{N_0} \left( \frac{\text{PARTICLE REFRACTIVE INDEX}}{\text{REFRACTIVE INDEX OF MEDIUM}} \right) \]

\[ \frac{\lambda}{d} \]

- BIG PARTICLE SCATTERING SHOULD BE \( Z A c \) BUT NOT REALLY TRUE
**Very Small Particle**

\[ Q_a > 1 \quad \xi = \frac{P_0 a}{4 \pi r^2 E_0} \]

Can be \( > 1 \) → violates Planck's law, but Planck says his law is not valid in this case.

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- → ▲

Detector

+ Detector should be "small", so as not to pick-up side scattered power

Power measured @ detector

\[ U = I_i (A_e - C_e) \]

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**Rayleigh Scattering** \((x << 1)\)

\[ Q_s = \frac{8}{3} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 x^4 \sim \frac{r^4}{\lambda^4} \quad C_s \sim r^6 \sim \lambda^2 \]

\[ Q_e = 4 \Im \left( \frac{m^2 - 1}{m^2 + 2} \right) x \sim \frac{r}{\lambda} \quad C_a \sim r^3 \sim \lambda^4 \]

\[ \sim Q_a \]

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**Induced Polarization**

\[ \overline{P} = \left( \frac{\varepsilon - 1}{\varepsilon + 2} \right) \frac{r^3}{\varepsilon} E_{\text{ind}} \]

\[ \frac{\overline{P}}{D} \]
Rayleigh Phase Function

\[ \phi(\theta) = \frac{3}{4} \left( 1 + \cos^2 \theta \right) \]

Rayleigh - Gans Limit \( |m-1| \ll 1 \)

Geometric Limit \( X|m-1| \ll 1 \)

Rainbow phenomenon \( \Rightarrow \) related to phase function

\( H_2O \) particle

\[ \cos \theta_\ell = \sqrt{\frac{n^2 - 1}{n^2 \ell(n^2 + 2) - 1}} \]

\( n = 1.33 \), \( \theta_\ell = 42^\circ \)

Various angles scatter different wavelengths of light from the \( H_2O \) particle more strongly or weakly.
SYSTEM OF PARTICLES

\[ N_T = \frac{\text{PARTICLE \# DENSITY}}{\text{VOL.}} \]

UNIFORM SIZE SCATTERING COEFF.

\[ 0_s = C_s N_T - \frac{1}{m} \]

ABS. COEF. \[ \chi = C_a N_T \]

EXT. COEF \[ \psi = C_e N_T = 0_s + \chi \]