Open Cycle

Jet engine

as a side note: if the net work were converted to velocity via a nozzle (jet engine) the relationships would be

\[ W_{\text{net\.dot}} = W_{t\.dot} - W_{c\.dot} \]

determines state 4 out of turbine at \( p_4 > p_1 \) atmosphere is state 5

\[ T_4 \] determined from equation for net work

\[ w_{\text{net}} = c_p \left( T_3 - T_4 \right) \]

could determine \( p_4 \) from

\[ \frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^\frac{\gamma - 1}{\gamma} \]

determine \( T_5 \) from

\[ \frac{T_5}{T_3} = \left( \frac{p_5}{p_3} \right)^\frac{\gamma - 1}{\gamma} \]

or...

\[ \frac{T_5}{T_4} = \left( \frac{p_5}{p_4} \right)^\frac{\gamma - 1}{\gamma} \]

nozzle anlysis:

First law, \( Q = W = 0 \)

\[ h_4 = h_5 + \frac{V^2}{2} \]

determines \( V \), thrust from momentum change

combustor ...

\[ 1 = \text{atmosphere} \ldots \]

adiabatic combustion \( Q = W = 0 \)

\[ 0 = H_{R2} - H_{P3} \]

\[ 0 = \text{Enthalpy of reactants at combustor inlet, compressor outlet} \]

- Enthalpy of products out of combustor - first law

rewrite using LHV ...

\[ 0 = H_{R2} - H_{R0} - \left( H_{P3} - H_{P0} \right) + \text{LHV} \]

rewrite using specific enthalpy and mass flows ... on a per unit mass flow of fuel ...

\[ 0 = h_{f2} - h_{f0} + \frac{m_{a\.dot}}{m_{f\.dot}} \left( h_{a2} - h_{a0} \right) - \left( 1 + \frac{m_{a\.dot}}{m_{f\.dot}} \right) \left( h_{p3} - h_{p0} \right) + \text{LHV} \]

to account for incomplete combustion introduce combustion efficiency ...

\[ \text{only obtain } \eta_{\text{comb}} \cdot \text{HV} \]

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Given

\[ 0 = h_{f2} - h_{f0} + \frac{m_{a\_dot}}{m_{f\_dot}} (h_{a2} - h_{a0}) - \left(1 + \frac{m_{a\_dot}}{m_{f\_dot}}\right) (h_{p3} - h_{p0}) + \eta_{comb}\cdot LHV \]

can solve for \( m_{a\_dot}/m_{f\_dot} \)

\[ \frac{m_{a\_dot}}{m_{f\_dot}} = \frac{\eta_{comb}\cdot LHV + (h_{f2} - h_{f0}) - (h_{p3} - h_{p0})}{h_{p3} - h_{p0} - (h_{a2} - h_{a0})} \]

introduce average specific heat ...

\[ c_{p\_bar\_air} = \frac{h_{a2} - h_{a0}}{T_2 - T_0} \quad c_{p\_bar\_prod} = \frac{h_{p3} - h_{p0}}{T_3 - T_0} \quad c_{p\_bar\_fuel} = \frac{h_{f2} - h_{f0}}{T_2 - T_0} \]

\[ \frac{m_{a\_dot}}{m_{f\_dot}} = \frac{\eta_{comb}\cdot LHV + c_{p\_bar\_fuel}(T_2 - T_0) - c_{p\_bar\_prod}(T_3 - T_0)}{c_{p\_bar\_prod}(T_3 - T_0) - c_{p\_bar\_air}(T_2 - T_0)} \]

or ... inverting

\[ \frac{m_{f\_dot}}{m_{a\_dot}} = \frac{c_{p\_bar\_prod}(T_3 - T_0) - c_{p\_bar\_air}(T_2 - T_0)}{\eta_{comb}\cdot LHV + c_{p\_bar\_fuel}(T_2 - T_0) - c_{p\_bar\_prod}(T_3 - T_0)} \]

gas turbine efficiency efficiency dividing by \( m_{a\_dot} \)

\[ \eta = \frac{W_{net\_dot}}{m_{f\_dot}\cdot LHV} = \frac{W_{t\_dot} + W_{c\_dot}}{m_{f\_dot}\cdot LHV} = \left(1 + \frac{m_{f\_dot}}{m_{a\_dot}}\right) \cdot \frac{c_{p\_bar\_prod}(T_3 - T_4) - c_{p\_bar\_air}(T_2 - T_1)}{m_{f\_dot}} \cdot \frac{LHV}{m_{a\_dot}} \]

\[ SFC = \frac{\text{kg fuel}}{\text{hr}} = \frac{\text{kg}}{\text{kW\cdot hr}} = \frac{\text{lb}}{\text{hp\cdot hr}} \]

not equality ...

\[ SFC = \frac{m_{f\_dot}}{W_{net\_dot}} = \frac{m_{f\_dot}}{W_{net\_dot}} \cdot \frac{LHV}{LHV} = \frac{1}{\eta\cdot LHV} \]
Open cycles have similar alternatives to closed analysis would be similar as well so not repeated here

**Open Cycle Regenerative (Recouperative)**

![Diagram of Open Cycle Regenerative (Recouperative)](image)

\[
\text{W}_{\text{dot.net}} = \text{W}_{\text{t, dot}} + \text{W}_{\text{c, dot}}
\]

N.B. cycle is drawn closed from state 6 to 1 but is taking place in atmosphere

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Intercooled Regenerative (Recouperative) Cycle

Rolls Royce WR-21 is an example see links

static data for plot

T-s diagram

- irreversible cycle
- irreversible, heat exchanger maximum
- regeneration inlet temperature irreversible
Various thermodynamic models can be used for analysis of products of combustion:

1. Single gas model
   - Perfect gas, constant $c_p$ (1 kJ/kg*K close enough), $\gamma = 1.4$

2. Two gas model
   - a) Perfect gas - air for compression, $c_p = 1.0035$ kJ/kg*K, $\gamma_a = 1.4$
   - b) Perfect gas combustion products; $c_{pp} = 1.13$ kJ/kg*K, $\gamma_p = 1.3$

3. Tabulated data (e.g. Keenan & Kaye Gas Tables)
   - Property data for air:
     - Table 1: Air at low pressure: $T$ deg F abs, $t$ deg F, $h$, $pr$, $u$, $v_r$, $\phi$
     - Table 2: Air at low pressures: $T$, $t$, $c_p$, $c_v$, $k = c_p/c_v$, $a$, $G_{max}/p_i$, $\mu$, $\lambda$, $Pr$
   - Table 3: $R \log N$ for air
   - Table 4: Products - 400% Theoretical Air (for One Pound Mole)
   - Table 5: Products - 400% Theoretical Air (for One Pound Mole) fuel data
   - Table 6: Products - $R_{bar} \log e N + 4.57263n$
   - Table 7: Products - 200% Theoretical Air (for One Pound Mole)
     - etc. data for oxygen, hydrogen, carbon monoxide, dioxide etc.

\[ T = \text{deg F abs} \quad c_p = \text{specific heat at constant pressure} \]
\[ t = \text{deg F} \quad c_v = \text{specific heat at constant volume} \]
\[ h = \text{enthalpy per unit mass} \quad G = \text{flow per unit area or mass velocity} \]
\[ p_r = \text{relative pressure} \quad k = c_p/c_v \]
\[ u = \text{internal energy per unit mass} \quad p = \text{pressure} \]
\[ \nu_r = \text{relative volume} \quad Pr = \text{Prandtl number} = c_p\mu/\lambda. \]
\[ \phi = \frac{1}{T_0} \int_{T_0}^{T} \frac{c_p}{T} dT \]
\[ \lambda = \text{thermal conductivity} \quad \mu = \text{viscosity} \]

Notes: Appendix (Sources and methods)
- "...calculated for one particular composition of the hydrocarbon fuel, it has been shown that it represents with high precision the properties of the products of combustion of fuels of a wide range of composition - all for 400% theoretical air." page 205 bottom
- Problems involving intermediate mixtures to Table B: can be solved by interpolation based on theoretical air
  or ... extrapolated to 100% for products is valid except for effects of dissociation

\[
\text{Table}_B = \begin{pmatrix}
\text{products} & \%\text{theor} & \%\text{theor} & \%\text{theor} & \text{air and water_vapor} \\
\text{Number} & \text{air} & \text{fuel} & \text{fuel} & \text{mass, \%_water} \\
1 & \text{inf} & 0 & 0 & 0 \\
4 & 400 & 25 & 14 & 6.7 \\
7 & 200 & 50 & 28 & \\
\end{pmatrix}
\]
4. Polynomial equations
   - example in combustion example $c_p = f(\theta)$

   isentropic process

   \[ ds = c_{po} \frac{dT}{T} - R \cdot \frac{dp}{p} \]  (7.21) in gas relationships

   \[ \frac{dp}{p} = \frac{1}{R} \cdot c_{po} \frac{dT}{T} \]

   \[
   \ln \left( \frac{p_1}{p_2} \right) = \frac{1}{R} \int_{T_2}^{T_1s} \frac{c_p}{T} \, dT
   \]

   \[
   \frac{1}{R} \int_{T_2}^{T_1s} \frac{c_p}{T} \, dT
   \]

   \[
   \frac{p_1}{p_2} = e
   \]

**High Temperature Gas Turbines**

Advantages:
- high efficiency - low specific fuel consumption
- high specific horsepower - small size and weight

Disadvantages:
- materials strength problems (Creep) see separate notes re: creep
- corrosion

Solutions:
- better materials
- blade and combustor cooling
- ceramic materials


**blade cooling**

Compressed air ducted into stationary AND rotor blades. Temperature reduced by:
- convective heat transfer
- transpiration (evaporation of water from surface)
- film

**Nominal data for plot**

**Cooling effectiveness (nominal)**

\[
\text{cooling\_effectiveness} = \frac{T_{\text{blade\_gas}} - T_{\text{blade\_metal}}}{T_{\text{blade\_gas}} - T_{\text{cooling\_air}}}
\]
Ceramic materials

Examples: silicon nitride, silicon carbide

Can be pressed, bonded and/or sintered to produce complete rotor system

\[
\begin{array}{cccc}
\text{tensile strength} & \text{MPa} & \text{ksi} & \text{MPa} & \text{ksi} \\
\text{Si}_3\text{N}_4 & 552 & 80 & 172 & 25 \\
\text{Si-C} & 193 & 28 & 138 & 20 \\
\end{array}
\]
Intercooled Regenerative Gas Turbine
typically two spool design

\[ W\_\text{net} = W\_turb + W\_comp \]

\[ m\_\text{fuel} \]

\[ \text{powers ... (review) reversible} \]

\[ \text{LP}\_\text{comp} = -m\_\text{air} \cdot (h_2 - h_1) \]

\[ \text{HP}\_\text{comp} = -m\_\text{air} \cdot (h_4 - h_3) \]

\[ \text{HP}\_\text{turb} = (m\_\text{air} + m\_\text{fuel}) \cdot (h_6 - h_7) \]

\[ \text{LP}\_\text{turb} = (m\_\text{air} + m\_\text{fuel}) \cdot (h_7 - h_8) \]

\[ \text{Power}\_\text{turb} = (m\_\text{air} + m\_\text{fuel}) \cdot (h_8 - h_9) \]

\[ Q_{H\_\text{dot}} = (m\_\text{air} + m\_\text{fuel}) \cdot (h_6 - h_5) \]

\[ w\_\text{dot}_{LP\_\text{comp}} = -w\_\text{dot}_{LP\_\text{turb}} \]

\[ w\_\text{dot}_{HP\_\text{comp}} = -w\_\text{dot}_{HP\_\text{turb}} \]

\[ \frac{m\_\text{air}}{m\_\text{fuel}} \]

from combustion analysis
Marinization

Problems:
1. sea water droplets in air (inlet)
2. sea water in fuel
3. coupling to the propeller
4. long ducting

Solutions:
1. sea water in air
   1. design of inlet - demisters to remove droplets
      demisters
      wire mesh
      inertial separation
   2. select corrosion resistant materials
   3. surface treatment of components - plating to improve corrosion resistance
   4. water washing and abrasive cleaning
2. sea water in fuel
   1. treat to remove sodium
3. coupling to propeller (later)
4. long ducting
   inlet and exit pressures reduce the pressure ratio across turbine
   reduction in power
   increase in fuel consumption
   additional effect from inlet density

\[ p \cdot v = R \cdot T \quad \Rightarrow \quad \frac{p}{\rho} = R \cdot T \quad \rho = \frac{p}{R \cdot T} \]
T-s diagram

similar effect for T inlet > nominal
cycle will walk up p₁ curve

normally cannot increase T₄ to account for these losses

other issues/topics

Materials
  coatings
  use of titanium

fuel treatment
  sodium - bad - corrosion from products
  remove by washing
  add agents such as demulsifiers
  water combines with sodium - remove by centrifuge
  vanadium - in Bunker C combines with sulfur - creates corrosive combustion products
  GE fro example has an additive to modify ash to prevent adhering to blades

problem 3 above: coupling to propeller
  1. Controllable Reversible Pitch Propeller (CRP)
  2. reversing gearbox
  3. electric drive
  4. reversing turbine

    concentric opposite direction direction blade annuli
Brayton cycle applied to turbocharging reciprocating engines

\[
\begin{align*}
\text{comp} &= -m_{\text{air dot}} (h_2 - h_1) \\
\text{turb} &= (m_{\text{air dot}} + m_{\text{fuel dot}}) (h_3 - h_4)
\end{align*}
\]

\[
\begin{align*}
\frac{w_{\text{dot comp}} + w_{\text{dot turb}}}{m_{\text{dot}}} &= 0 = (m_{\text{air dot}} + m_{\text{fuel dot}}) (h_3 - h_4) - m_{\text{air dot}} (h_2 - h_1) \\
h_2 - h_1 &= \left(1 + \frac{m_{\text{fuel dot}}}{m_{\text{air dot}}}\right) (h_3 - h_4)
\end{align*}
\]

\[p_3\text{ may be } >\text{ or } < p_2\text{ depending on what happens in engine}\]

**combined cycles - gas turbine and Rankine - or other**

maximum available power from \(T_4 \rightarrow T_5\)

\[
\begin{align*}
\left(\frac{w_{\text{dot rev}}}{m_{\text{dot}}}\right)_{\text{max}} &= \psi_4 - \psi_5 = h_4 - T_0 s_4 - (h_5 - T_0 s_5) = h_4 - h_5 - T_0 (s_4 - s_5) \\
\text{second law ... } T \cdot ds &= dh - v \cdot dp \\
\text{assuming } c_{\text{pp}}\text{ constant } ds &= \frac{dh}{T} = \frac{c_{\text{pp}} dT}{T} \Rightarrow s_4 - s_5 = c_{\text{pp}} \ln \left(\frac{T_4}{T_5}\right)
\end{align*}
\]
example LM 2500

\[ T_4 := 825 \text{ K} \]
\[ T_0 := 300 \text{ K} \]
\[ \text{GT\_power} := 330 \frac{\text{kW}}{\text{kg}} \]
\[ kJ := 1000\text{J} \]

\[ 1 < c_{p\_prod} < 1.33 \]

\[ c_{p\_prod} := 1.08 \frac{\text{kJ}}{\text{kg\cdotK}} \]

\[ T_5 := \begin{pmatrix} 500 \\ 400 \\ 300 \end{pmatrix} \]

\[ T_4 - T_5 = \begin{pmatrix} 325 \\ 425 \\ 525 \end{pmatrix} \quad T_0 \cdot \ln \left( \frac{T_4}{T_5} \right) = \begin{pmatrix} 150.233 \\ 217.176 \\ 303.48 \end{pmatrix} \]

\[ \left( \frac{w\_dot\_rev}{m\_dot} \right)_{\text{max}} = W\_m\_dot\_max \]

\[ W\_m\_dot\_max := c_{p\_prod} \left( T_4 - T_5 - T_0 \cdot \ln \left( \frac{T_4}{T_5} \right) \right) \cdot K \]

\[ W\_m\_dot\_max = \begin{pmatrix} 188.749 \\ 224.45 \\ 239.241 \end{pmatrix} \frac{\text{kJ}}{\text{kg}} \]

\[ \frac{W\_m\_dot\_max}{\text{GT\_power}} = \begin{pmatrix} 0.572 \\ 0.68 \\ 0.725 \end{pmatrix} \]