Helical Gears

advantages ...
greater load capacity
smoother operation
less sensitivity to tooth errors
teeth are at angle to rotation, contact is a series of oblique lines with several lines in contact simultaneously. total length of contact varies as teeth mesh.

offset adjacent "strings" in involute generator concept on base cylinder by angle $\psi$

$$\begin{align*}
L &= \frac{2 \pi R_B}{\tan(\psi_B)} \\
R_B &= \text{base_radius} \\
\psi_B &= \text{base_helix_angle}
\end{align*}$$

develop normal at any radius on tooth by considering transverse and normal planes intersecting tooth at that point.

geometry development

(Xn, Yn, Zn), (Xn1, Yn1, Zn1), (X_line, Y_line, Z_line), (Xg, Yg, Zg), (X, Y, Z)

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point B ... point on gear for normal with helix (shown off gear)
point A ... point on radial line 0,0 to B perpendicular joining tangent
point D ... tangent point
point E ... point on plane perpendicular to tooth at B, connecting with (transverse) tangent point along $R_B$

\[
\tan(\phi_N) = \frac{AB}{AE} \quad \tan(\phi_T) = \frac{AB}{AD} \quad \cos(\psi) = \frac{AD}{AE}
\]

\[
\frac{AD \cdot AB}{AE \cdot AD} = \cos(\psi) \cdot \tan(\phi_T) = \tan(\phi_N) \quad \Rightarrow \quad \tan(\phi_T) = \frac{\tan(\phi_N)}{\cos(\psi)} \quad \text{or} \quad \phi_T = \arctan\left(\frac{\tan(\phi_N)}{\cos(\psi)}\right)
\]

other parameters; as in general gear ...

\[
P_t = \text{circular\_pitch\_transverse} = \frac{\pi \cdot D}{N_g} = \frac{\pi \cdot d}{N_p}
\]

\[
D = \text{diameter\_gear} \quad N_g = \text{number\_of\_teeth\_gear} \quad d = \text{diameter\_pinion} \quad N_p = \text{number\_of\_teeth\_pinion}
\]

considering an expanded view at any radius ...

3. Pitch \[ P_t = \text{circular\_pitch\_transverse} = \frac{\text{pitch\_circumference}}{\text{number\_of\_teeth}} = \frac{\pi \cdot D}{N_g} = \frac{\pi \cdot d}{N_p} \] \[ P_n = \text{circular\_pitch\_normal} = P_t \cdot \cos(\psi) \] \[ g = \text{gear} \quad p = \text{pinion} \]

\[ P_t = \text{diametral\_pitch\_transverse} = \frac{\text{number\_of\_teeth}}{\text{pitch\_diameter}} = \frac{N_g}{D} = \frac{N_p}{d} \]

so ... \[ P_t = \frac{\pi}{p} \text{ and } ... \quad P_n = \frac{\pi}{P_N} \]

N.B. above geometry \[ \tan(\psi) = \frac{P_t}{P_x} \]

\[ \Rightarrow \quad P_x = \frac{P_t}{\tan(\psi)} \]

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\[ P_x = \text{axial\_pitch} = P_t \cot(\psi) \]

\[ P_b = \text{base\_pitch\_transverse} = \frac{\text{base\_circumference}}{\text{number\_of\_teeth}} = \frac{\pi \cdot 2R_B}{N} = \frac{\pi \cdot 2R_G}{N} \cdot \frac{R_B}{R_G} \]

\[ R_G = \text{pitch\_radius} = \frac{D}{2} \]

from geometry way above ... 0.0, A, B, D

\[ \cos(\phi_t) = \frac{R_B}{R_G} \]

\[ \Rightarrow P_b = \frac{\pi D}{N_g} \cdot \cos(\phi_t) = P_t \cdot \cos(\phi_t) \]

\[ P_{bN} = \text{base\_pitch\_normal} \]

consider geometry at left which is above brought down to base radius

\[ P_x \text{ is common, not dependent on radius ... } P_{bN} \text{ forms altitude of triangle with sides } P_x \text{ and } P_b \text{ and base } \sqrt{(...) \text{ calculate area} \]

\[ \text{area} = \frac{1}{2} \cdot \text{base-altitude} = \frac{1}{2} \sqrt{P_b^2 + P_x^2 \cdot P_{bN}} \]

and ...

\[ \text{area} = \frac{1}{2} \cdot P_x \cdot P_b \quad \text{as ... is right triangle ...} \]

\[ \Rightarrow \sqrt{P_b^2 + P_x^2} \cdot P_{bN} = P_x \cdot P_b \quad \text{and ...} \quad P_{bN} = \frac{P_x \cdot P_b}{\sqrt{P_b^2 + P_x^2}} \]

also ...

\[ P_{bN} = \frac{P_x}{\sqrt{P_b^2 + P_x^2}} \cdot P_b = \cos(\psi_B) \cdot P_t \cdot \cos(\phi_t) \]

from figure and above ..

also ...

\[ P_{bN} = P_t \cdot \cos(\psi) \cdot \cos(\phi_n) \]

not shown here ...

\[ P = \text{diametral\_pitch\_transverse} = \frac{N_g}{D} \quad \text{shown above ...} \]

\[ P_t = \frac{\pi}{P} \quad \text{and ...} \quad P_n = \frac{\pi}{P_n} \]

\[ P = \text{diametral\_pitch\_normal} = \frac{\pi}{P_n} = \frac{\pi}{P_t \cdot \cos(\psi)} = \frac{P}{\cos(\psi)} \]

**additional note on tooth loading ...**

\[ \frac{\text{hp}}{\left( \frac{2\pi}{\text{min}} \right) \cdot 1\text{\_in}} = 126051 \text{\_lbf\_in} \]

\[ W_t = \text{tangential\_tooth\_load} = 126050 \cdot \frac{\text{HP}}{\text{RPM}_p \cdot d} = 126050 \cdot \frac{\text{HP}}{\text{RPM}_g \cdot D} \]

\[ W_T = \text{total\_tooth\_load\_transverse\_plane} = \frac{W_t}{\cos(\phi_t)} \]

\[ W_n = \text{tangential\_tooth\_load\_normal\_plane} = \frac{W_t}{\cos(\psi)} \]

\[ W_N = \text{total\_tooth\_load\_normal\_plane} = \frac{W_t}{\cos(\phi_n) \cdot \cos(\psi)} \]