2.626/2.627 Roadmap

You Are Here

Fundamentals

Theme 1
Theme 2
Theme 3
Theme 4
Theme 5
Theme 6

Tech A Tech B Tech C Tech D Tech E
2.626/2.627: Fundamentals

*Every* photovoltaic device must obey:

\[
\text{Conversion Efficiency } (\eta) \equiv \frac{\text{Output Energy}}{\text{Input Energy}}
\]

For most solar cells, this breaks down into:

Inputs

- Solar Spectrum
- Light Absorption
- Charge Excitation

Outputs

- Charge Drift/Diffusion
- Charge Separation
- Charge Collection

\[
\eta_{\text{total}} = \eta_{\text{absorption}} \times \eta_{\text{excitation}} \times \eta_{\text{drift/diffusion}} \times \eta_{\text{separation}} \times \eta_{\text{collection}}
\]
Liebig’s Law of the Minimum

\[ \eta_{\text{total}} = \eta_{\text{absorption}} \times \eta_{\text{excitation}} \times \eta_{\text{drift/diffusion}} \times \eta_{\text{separation}} \times \eta_{\text{collection}} \]

Diode: Essence of Charge Separation

- What is a diode?
- How is it made?
- Why care about diodes?
Diode: Essence of Charge Separation

Courtesy of Adrio Communications Ltd. Used with permission.

Learning Objectives: Diode

1. **Describe how conductivity of a semiconductor can be modified by the intentional introduction of dopants.**

2. Draw pictorially, with fixed and mobile charges, how built-in field of pn-junction is formed.

3. Current flow in a *pn*-junction: Describe the nature of drift, diffusion, and illumination currents in a diode. Show their direction and magnitude in the dark and under illumination.

4. Voltage across a *pn*-junction: Quantify the built-in voltage across a *pn*-junction. Quantify how the voltage across a *pn*-junction changes when an external bias voltage is applied.

5. Draw current-voltage (I-V) response, recognizing that minority carrier flux regulates current.
Dopant Atoms

Periodic Table

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<tr>
<td>Tl</td>
<td>Pb</td>
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<td>Po</td>
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n-doped silicon

e- extra electron

group V atom

p-doped silicon

h+ missing electron (hole)
group III atom

http://pvcdrom.pveducation.org/

Courtesy of PVCDROM. Used with permission.
Carrier binding energy to a shallow (hydrogenic) dopant atom:

\[ E = E_H \left( \frac{m^*}{m_e} \right) \frac{1}{\varepsilon^2} = (13.6 \text{ eV}) \cdot \left( \frac{m^*}{m_e} \right) \frac{1}{\varepsilon^2} \]
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Carrier Binding Energy to Shallow Dopant Atoms

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Gauss’ Law: Review

Spatially variant fixed charge creates an electric field:

\[
\frac{d \xi}{dx} = \frac{\rho}{\varepsilon}
\]

\(\xi\) = electric field
\(\rho\) = charge density
\(\varepsilon\) = material permittivity

Example: Capacitor

\[\nabla \cdot \xi = \frac{\rho}{\varepsilon}\]
Gauss’ Law: Review

Spatially variant fixed charge creates an electric field:

\[ \frac{d\xi}{dx} = \frac{\rho}{\varepsilon} \]

\( \xi = \text{electric field} \)
\( \rho = \text{charge density} \)
\( \varepsilon = \text{material permittivity} \)

Drift Current: Net charge moves parallel to electric field

\[ J_h = q\mu_h p\xi \]
\[ J_e = q\mu_e n\xi \]

Described by Drift Equation

From: PVCDROM

Courtesy of PVCDROM. Used with permission.
Diffusion: Review

Described by Fick's Law

\[ J_h = -qD_h \frac{dp}{dx} \]

\[ J_e = qD_e \frac{dn}{dx} \]

In this animation, 1/4 of the carriers move to the right, 1/4 to the left and the remainder stay put (move up or down).

Carriers in each location at scattering event #1:
- 4
- 8
- 4

Carriers in each location at scattering event #2:
- 1
- 4
- 6
- 4
- 1

Carriers in each location at scattering event #3:
- 2
- 4
- 4
- 4
- 2

Carriers in each location at scattering event #4:
- 1
- 2
- 3
- 4
- 3
- 2
- 1

Process continues until a uniform concentration results.

From PVCDROM

Courtesy of PVCDROM. Used with permission.
Recall the Checker Board Example
Let’s imagine the n- and p-type materials in contact, but with an imaginary barrier in between them.
How a pn-junction comes into being

With the P and N type materials separated the carriers diffuse around randomly.

Courtesy of PVCDROM. Used with permission.
When that imaginary boundary is removed, electrons and holes diffuse into the other side.
Eventually, the accumulation of like charges \([ (h^+ + P^+) \text{ or } (e^- + B^-) ] \) balances out the diffusion, and steady state condition is reached.

How a pn-junction comes into being

Courtesy of PVCDROM. Used with permission.
How a pn-junction comes into being

The net charge can be approximated as shown above.

Courtesy of PVCDROM. Used with permission.

The net charge can be approximated as shown above.
How a pn-junction comes into being

\[ \frac{d \xi}{dx} = \frac{\rho}{\varepsilon} \]

\[ \frac{d \psi}{dx} = -\xi \]

\[ E = q \psi \]

\[ \psi_o - V_A \]

\[ q(\psi_o - V_A) \]

Courtesy of PVCDROM. Used with permission.
Summary of Current Understanding

1. When light creates an electron-hole pair, a $pn$-junction can separate the positive and negative charges because of the built-in electric field.

2. This built-in electric field is established at a $pn$-junction because of the balance of electron & hole drift and diffusion currents.
In-Class Exercise
**pn-junction, under dark conditions**

<table>
<thead>
<tr>
<th>Model Circuit</th>
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<td><img src="image" alt="Drift Diagram" /></td>
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<td>P - + N</td>
<td>P N</td>
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**Band Diagram**

- E
- p-type
- n-type
- e⁻ diffusion:
- e⁻ drift:

**Tasks:**

1. Draw band diagram (electron energy as a function of position).
2. Draw relative magnitudes of electron drift and diffusion currents.

**I-V Curve**

- I
- V

2.626/2.627 Lecture 5 (9/22/2011)
### pn-junction, under dark conditions

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**Key Terms**
- **Band Diagram**: Illustrates the energy levels of electrons and holes in a semiconductor.
- **I-V Curve**: Represents the current-voltage relationship of a semiconductor device.
- **p-type**: Semiconductors with holes as majority charge carriers.
- **n-type**: Semiconductors with electrons as majority charge carriers.

**Concepts**
- **Diffusion**: Movement of charge carriers due to concentration gradient.
- **Drift**: Movement of charge carriers due to an applied electric field.

**Formula**
- **E**: Energy level of electrons.
- **V**: Voltage.
### pn-junction, under dark conditions

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- **h⁺ diffusion:** →
- **h⁺ drift:** ←
- **e⁻ diffusion:** →
- **e⁻ drift:** ←

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**Band Diagram**

- E
- p-type
- n-type
- e⁻ diffusion: [Diagram]
- e⁻ drift: [Diagram]

**Tasks:**
1. Represent a voltage bias source (e.g., battery) on the model circuit diagram. Ensure that positive and negative terminals of the battery are pointing in the correct directions.

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- e⁻ diffusion: ← →
- e⁻ drift: → ←
**pn-junction, under dark conditions**

### Tasks:
1. Draw energy band diagrams, under forward and reverse bias (in the dark).
2. Draw relative magnitudes of electron drift and diffusion currents.

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**pn-junction, under dark conditions**

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5. Draw current-voltage (I-V) response, recognizing that minority carrier flux regulates current.
Carrier Motion

Under equilibrium conditions in a homogeneous material: Individual carriers constantly experience Brownian motion, but the net charge flow is zero.

To achieve net charge flow (current), carriers must move via diffusion or drift.
Diffusion

Described by Fick’s Law

\[ J_h = -qD_h \frac{dp}{dx} \]
\[ J_e = qD_e \frac{dn}{dx} \]
Drift Current

Described by Drift Equation

\[ J_h = q \mu_h p \xi \]
\[ J_e = q \mu_e n \xi \]

From PVCDROM

Courtesy of PVCDROM. Used with permission.
Current Density Equations

\[ J_e = q\mu_n n \xi + qD_e \frac{dn}{dx} \]
\[ J_h = q\mu_h p \xi - qD_h \frac{dp}{dx} \]

Dominates when \( \xi \) is large
Dominates when \( \xi \) is small

Einstein Relationships:
Relation between drift and diffusion:

\[ D_e = \left( \frac{kT}{q} \right) \mu_n \]
\[ D_h = \left( \frac{kT}{q} \right) \mu_p \]
What’s $\xi$?

From differential form of Gauss’ Law (a.k.a. Poisson’s Equation):

$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

$\rho = \text{charge density}$

$\varepsilon = \text{material permittivity}$

We know the charge density is:

$$\rho = q(p - n + N_D^+ - N_A^-)$$

$$\rho \approx q(p - n + N_D - N_A)$$

$N_D^+ = \text{ionized donor concentration}$

$N_A^- = \text{ionized acceptor concentration}$

Assuming all dopants are ionized at room temperature

In summa:

$$\frac{d\xi}{dx} = \frac{q}{\varepsilon} (p - n + N_D - N_A)$$
What’s $\xi$?

\[
\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}
\]

\[
dx = -\xi
\]

\[
E = q\psi
\]

Courtesy of PVCDROM. Used with permission.
Continuity Equations

rate entering - rate exiting = \frac{A}{q} \left\{ J_e(x) - \left[ J_e(x + \delta x) \right] \right\}

= \frac{A}{q} \frac{dJ_e}{dx} \delta x

rate of generation - rate of recombination = A \delta x (G - U)

For electrons:
\[ \frac{1}{q} \frac{dJ_e}{dx} = U - G \]

For holes:
\[ \frac{1}{q} \frac{dJ_h}{dx} = -(U - G) \]
System of Equations Describing Transport in Semiconductors

\[ J_e = q\mu_n n\xi + qD_e \frac{dn}{dx} \]

\[ J_h = q\mu_h p\xi - qD_h \frac{dp}{dx} \]

\[ \frac{d\xi}{dx} = \frac{q}{\varepsilon} \left( p - n + N_D - N_A \right) \]

\[ \frac{1}{q} \frac{dJ_e}{dx} = U - G \]

\[ \frac{1}{q} \frac{dJ_h}{dx} = -(U - G) \]
Possible to Solve Analytically?

No! Coupled set of non-linear differential equations.

Must solve numerically (e.g., using computer simulations)...

...or make series of approximations to solve analytically.
Learning Objectives: Diode

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2. Draw pictorially, with fixed and mobile charges, how built-in field of pn-junction is formed.
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New Concept: **Chemical Potential**

Band Diagram (E vs. x)
New Concept: **Chemical Potential**

At absolute zero, no conductivity (perfect insulator).

Band Diagram (E vs. x)

Courtesy of PVCDROM. Used with permission.
New Concept: **Chemical Potential**

At $T > 0$ K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. x)

Courtesy of PVCDROM. Used with permission.
New Concept: *Chemical Potential*

At $T > 0$ K, some carriers are thermally excited across the bandgap.

![Band Diagram (E vs. x)](image)

Courtesy of PVCDROM. Used with permission.
New Concept: *Chemical Potential*

At $T > 0$ K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. $x$)

“Intrinsic” Carriers ($n_i$)

 Courtesy of [PVCDROM](http://pvcdrom.com). Used with permission.
New Concept: Chemical Potential

At $T > 0$ K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. x)

- The chemical potential describes the average energy necessary to add or remove an infinitesimally small quantity of electrons to the system.
- In a semiconductor, the chemical potential is referred to as the “Fermi level.”

Courtesy of PVCDROM. Used with permission.
We assume: All dopants are ionized!
Voltage Across a \textit{pn}-Junction

\begin{figure}
\centering
\begin{tikzpicture}
\draw[->] (0,0) -- (8,0) node[anchor=north] {Distance, $x$};
\draw[->] (0,-1) -- (0,4) node[anchor=east] {Energy, eV};
\draw[blue, dashed] (0,2) -- (8,2) node[anchor=west] {$Fermi$ Level, $E_F$};
\draw (0,2.5) -- (4,2.5) node[anchor=east] {$p$-type};
\draw (4,2.5) -- (4,1.5) node[anchor=east] {Transition region};
\draw (4,1.5) -- (8,1.5) node[anchor=east] {$n$-type};
\end{tikzpicture}
\end{figure}
Voltage Across a *pn*-Junction

\[ q \psi_0 = E_g - (E_F - E_V) - (E_C - E_F) \]

\[ = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Built-in *pn*-junction potential a function of dopant concentrations.
Derivation

\[ q \psi_\circ = E_g - (E_F - E_V) - (E_C - E_F) \]

\[ = E_g - kT \ln \left( \frac{N_V}{N_A} \right) - kT \ln \left( \frac{N_C}{N_D} \right) \]

\[ = E_g - kT \ln \left( \frac{N_C N_V}{N_A N_D} \right) \]

\[ \psi_\circ = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Built-in pn-junction potential a function of dopant concentrations.
Voltage Across a \textit{pn}-Junction

\[ q \psi_o = E_g - (E_F - E_V) - (E_C - E_F) \]

\[ = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Built-in \textit{pn}-junction potential a function of dopant concentrations.
Voltage Across a **Biased** $pn$-Junction

\[ q(\psi_o - V_A) = k_b T \ln\left( \frac{N_A N_D}{n_i^2} \right) - V_A \]
Effect of Bias on Width of Space-Charge Region

\[ q \psi_o = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]
Effect of Bias on Width of Space-Charge Region

\[ q(\psi_o - V_A) = k_b T \ln \left( \frac{N_A N_D}{n_i^2} \right) - V_A \]
**pn-junction, under dark conditions**

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- **I-V Curve**
  - ![I-V Curve](image)
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  - ![I-V Curve](image)
Learning Objectives: Diode

1. Describe how conductivity of a semiconductor can be modified by the intentional introduction of dopants.

2. Draw pictorially, with fixed and mobile charges, how built-in field of pn-junction is formed.

3. Current flow in a $pn$-junction: Describe the nature of drift and diffusion currents in a diode in the dark. Show their direction and magnitude under neutral, forward, and reverse bias conditions.

4. Voltage across a $pn$-junction: Quantify the built-in voltage across a $pn$-junction. Quantify how the voltage across a $pn$-junction changes when an external bias voltage is applied.

5. **Draw current-voltage (I-V) response, recognizing that minority carrier flux regulates current.**
Approximation 1: Device can be split into two types of region: quasi-neutral regions (space-charge density is assumed zero) and the depletion region (where carrier concentrations are small, and ionized dopants contribute to fixed charge).
Width of space charge region

\[ W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q} (\psi_o - V_a) \cdot \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} \]
Width of space charge region

\[ W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q} (\psi_o - V_a) \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} \]

\( p = p_{p0} \approx N_A \)

\( n = n_{n0} \approx N_D \)

\( n = n_{p0} \approx n_i^2/N_A \)

\( p = p_{n0} \approx n_i^2/N_D \)

\( \psi_o \)

\( V_a \)

\( N_A \)

\( N_D \)

\( \varepsilon \)

\( q \)

NB: Actually \( \varepsilon * \varepsilon_o \), where \( \varepsilon_o \), the vacuum permittivity, is 8.85x10^{-12} \text{ F/m} \text{ or } 5.53x10^7 \text{ e/(V*m)} \)
Capacitance

\[ C = \frac{\varepsilon A}{W} \]

Device capacitance

Distance, \( x \)

pn-junction area

\[ p = p_{p0} \approx N_A \]

\[ n = n_{p0} \approx \frac{n_i^2}{N_A} \]

\[ n = n_{n0} \approx N_D \]

\[ p = p_{n0} \approx \frac{n_i^2}{N_D} \]
When one side of the pn-junction is heavily doped, the capacitance reduces to this expression:

\[
\frac{C}{A} = \sqrt{\frac{q \varepsilon N}{2(\psi_0 - V_a)}}
\]
Pn-junction under zero bias

\[ p = p_{p0} \approx N_A \]

\[ n = n_{p0} \approx n_i^2 / N_A \]

\[ n = n_{n0} \approx N_D \]

\[ p = p_{n0} \approx n_i^2 / N_D \]
Pn-junction under **forward bias**

\[
\begin{align*}
\text{p-type} & : p = p_{p0} \approx N_A \\
\text{n-type} & : n = n_{n0} \approx N_D
\end{align*}
\]
Pn-junction under **forward bias**

\[ p = p_{p0} \approx N_A \]

\[ n = n_{n0} \approx N_D \]

At zero bias:

\[ p_{nb} = p_{n0} = p_{p0} \cdot \exp\left(-\frac{q \psi_0}{kT}\right) \approx \frac{n_i^2}{N_D} \]

\[ n_{pa} = n_{p0} = n_{n0} \cdot \exp\left(-\frac{q \psi_0}{kT}\right) \approx \frac{n_i^2}{N_A} \]
Current flow through the depletion region

\[ \frac{dn}{dx} = q \mu_n p \xi - q D_n \frac{dp}{dx} \]

For holes:

- Drift current
- Diffusion current

\[ p = p_{p0} \approx N_A \]
\[ n = n_{n0} \approx N_D \]
Current flow through the depletion region

For holes:

\[ \xi \approx \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \]
Current flow through the depletion region

Integrating...

\[ \psi_o - V_a = -\frac{kT}{q} \ln(p)|_b^a \]

\[ = \frac{kT}{q} \ln\left(\frac{p_{pa}}{p_{nb}}\right) \]
Current flow through the depletion region

**p-type**

\[ p = p_p0 \approx N_A \]

**Transition region**

\[ n_pa \]

**n-type**

\[ n = n_n0 \approx N_D \]

\[ p = p_{pa} = N_A + n_{pa} \]

Approximation 3: Only cases where minority carriers have a much lower concentration than majority carriers will be considered, i.e., \( p_{pa} \gg n_{pa}, n_{na} \gg p_{na} \)
Current densities

Calculate (diffusive) currents in quasi-neutral region:

\[ J_h = -qD_h \frac{dp}{dx} \]

... from previous slide ...

\[ J_h(x) = \frac{qD_h p_{n0}}{L_h} \left( e^{qV/kT} - 1 \right) e^{-x/L_h} \]

\[ J_e(x') = \frac{qD_e n_{n0}}{L_e} \left( e^{qV/kT} - 1 \right) e^{-x'/L_e} \]
Current densities

\[
\frac{1}{q} \frac{dJ_e}{dx} = U - G = - \frac{1}{q} \frac{dJ_h}{dx}
\]

Magnitude of the change in current across the depletion region:

\[
\delta J_e = |\delta J_h| = q \int_{-W}^{0} (U - G) dx \approx 0
\]

Key assumption: \( W \) is small compared to \( L_e \) and \( L_h \). Therefore, integral is negligible. It follows that the current \( J_e \) and \( J_h \) are essentially constant across the depletion region, as shown below.
Ideal Diode Equation

Since $J_e$ and $J_h$ are known at all points in the depletion region, we can calculate the total current:

$$J_{\text{total}} = J_e \bigg|_{x'=0} + J_h \bigg|_{x=0} = \left( \frac{qD_e n_p 0}{L_e} + \frac{qD_h p_n 0}{L_h} \right) \left( e^{qV/kT} - 1 \right)$$

This leads to the ideal diode law:

$$I = I_o \left( e^{qV/kT} - 1 \right) \quad \text{where}$$

$$I_o = A \left( \frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D} \right)$$
Key Point

• The IV response of a pn-junction is determined by changes in *minority carrier current* at the edge of the space-charge region.
Readings are strongly encouraged

• Green, Chapter 4
### pn-junction, under dark conditions

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Hands-On: Measure Solar Cell IV Curves