1. **A Binocular.** The simplified optical diagram of an arm of a binocular can be considered as a telescope, which consists of two lenses of focal lengths $f_1 = 25$ cm (objective) and $f_2 = 5$ cm (eyepiece). The normal observer's eye is intended to be relaxed and the nominal focal length of the eye lens is taken to be $f_{EL} = 40$ mm. **The first prism is placed 5 cm away from the objective and the two prisms are separated by 2 cm.**

   - **a) (10%)** In order to make the binocular compact, a pair of 45° prisms (5 cm wide) are used, each of them is designed for total internal reflection of incoming rays. Estimate the index of refraction needed to meet such a requirement under paraxial beam approximation.

   - **b-e) Assume the index of refraction of both prisms is 1.5.**

   - **b) (15%)** Please estimate the distance from the eyepiece to the back side of the second prism.

   - **c) (20%)** If two distant objects are separated by $10^{-3}$ rad to an observer with naked eye, how far apart (in units of length) will the images form on the observer's retina when the observer is using the binocular?

   - **d) (15%)** An aperture (D=3 cm) is placed inside the binocular, at a distance of 3 cm to the left of the eyepiece. Please locate the Entrance Window and Exit Window, and calculate the Field of View.

   - **e) (extra credit 10%)** Where is the optimum location of the observer's eye pupil in the configuration described by d)?
2. Reflection from a concave cavity. Figure 2 shows a reflective cavity made of concave mirrors, with light source $s$. The cavity is designed to reflect all rays leaving the source $s$ to a point $p$ along the long axis of the cavity.

![Diagram of a concave cavity with labeled points S(-h,0), P(h,0), and (x, y).]

a) (10%) Following Fermat’s principles, the optical path length from $s$ to $p$ on any point $(x, y)$ on the reflective cavity should be a constant. Please show such a constant is $2a$, the length of the long axis of the cavity.

b) (15%) Using Cartesian coordinates, please prove that any point $(x, y)$ on the reflective cavity must satisfy the following relationship:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where $2h = |SP|$ is the distance from $s$ to $p$, and $b = \sqrt{a^2 - h^2}$ is the length of the short axis of the cavity. Therefore, the cavity is an ellipse.

c) (15%) Assume the cavity is large enough so you can go in, and a small object is placed on the left side of the source $s$, as shown by the arrow in Figure 1. Use ray tracing, please locate the first reflected image of the object. Is it real or virtual?