1. **A Lloyd Mirror Interferometer.** The Lloyd mirror is a wavefront-splitting interferometer. It consists of a flat glass mirror that reflects a portion of wavefront that comes from a narrow slit. Another portion of the wavefront proceeds directly to the screen. The interference of the two wavefronts form a set of bright and dark fringes that can be measured on the screen.

In our problem, let’s assume the mirror is placed at the plane \( x = 0 \) and illuminated by a spherical wave originating from the slit at location \( (x_0, -z_0) \) (where \( x_0, z_0 > 0 \)). Using the paraxial approximation for a 1D spherical wave \((y=0)\),

\[
E(x, z) = E_0 \frac{\exp[ik(z + z_0)]}{i(z + z_0)} \exp\left[\frac{ik(x - x_0)^2}{2(z + z_0)}\right]
\]

**a)** (10%) The reflected wavefront from the mirror can be considered as spherical wave radiation from a virtual source. Using your knowledge from geometric optics, determine the origin of the virtual source that radiates such spherical wave.

**b)** (15%) The source illuminating the slit has a wavelength of 500nm in air. If the slit is positioned at \( x_0=1 \)mm above the flat mirror, and the screen is placed 1 meter away from the slit, please estimate the spacing of the fringes on the screen.

**c)** (15%) In order to find the relative amplitude and phase of the virtual source with respect to the original spherical wave, we have to consider the Fresnel equations for reflected waves.
For simplicity, let’s assume the E-field of source is polarized in the y-direction (S-polarization) and the slit is placed near the x=0 plane \( (x_0<<z_0, \text{so } \theta_i=\theta_r->90^\circ) \). Using these assumptions and based on the reflection coefficient for S-polarization at air-glass interface

\[
\frac{E_r}{E_{0i}} = \frac{k_{x,\text{air}} - k_{x,\text{glass}}}{k_{x,\text{air}} + k_{x,\text{glass}}}
\]

And \( k_{x,\text{air}} = k \sqrt{n_{\text{air}}^2 - \sin^2 \theta_i} \), \( k_{x,\text{glass}} = k \sqrt{n_{\text{glass}}^2 - \sin^2 \theta_i} \) \((n_{\text{air}}=1, n_{\text{glass}}=1.5)\).

Please estimate the relative amplitude (i.e. \(|r_s|\)) and phase (i.e. \(\arg(r_s)\)) of the virtual source when \( \theta_i \) approaches 90\(^\circ\).

*d) (10\%) Using the result you obtained from c), show that the fringe pattern is dark at x=0 on the screen.

2. Optical Fourier transforms.
   a) (20\%) Calculate the Fourier transform of the function:

   \[
g(x) = \frac{1}{2} \exp(-i\pi\alpha^2 x^2) \left[ 1 + \cos \left( \frac{2\pi x}{\Lambda} \right) \right]
   \]

   b) (30\%) Assuming \( \alpha = 10\text{mm}^{-1}, \Lambda = 10\mu\text{m} \), design an optical system which at its output plane creates an exact replica of the Fourier transform of the previous question. Additional constraints on your design are:

   1. Your system should be designed for spatially and temporally coherent illumination consisting of a plane wave at wavelength \( \lambda = 500\text{nm} \), incident on-axis.
   2. You may only use “standard” optical elements such as lenses, gratings, prisms, and free-space propagation.
   3. The spatial frequency component \( u = 50\text{mm}^{-1} \) should be mapped at distance of 3 cm away from the optical axis at the output plane.

GOOD LUCK!