2.72

Elements of Mechanical Design

Lecture 04: Fatigue
Schedule and reading assignment

Reading quiz

Announcements
- Shaft due date
- Shaft exercise
- Goodman diagram quiz (Tuesday)
- Shear-moment qualifying quiz (Tuesday)


Topics
- Discuss stiffness exercises
- Start fatigue

Reading
- None, for Tuesday, prep for quizzes in lab time (Given lounge, top of 35)

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Reading quiz
Discuss stiffness exercises

Answers

Intuition about stiffness

- Part
- Spindle
- Carriage-rail
- Etc…

Insight and perspective

Carriage bearing-rail

Combined Stiffness $K'$

$K'/2$
$K'/3$
$K'/4$
$K'/5$

0 1 2 3 4

$K'/K$
Shaft exercise
Are you on top of this!?
Fatigue part I
At what critical time in engineering history did fatigue become relevant?
Why does fatigue failure generate serious concern?
What type of warnings does one receive?
Fire plane wing failure

July 18, 2002 near Estes Park, Colorado

- Both crew members killed
- Delivered in July, 1945 to the U.S. Navy
- Logged 8000+ flight hours

Investigation

- NTSB found extensive fatigue
- Cracks hidden from view

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Comet airplane failures

BOAC Flight 781 crashes on 10 January 1954
- Concluded fire was most likely cause
- Resumed on 23 March 1954

Comet G-ALYY crashes on 8 April 1954
- Pressure tests revealed fatigue
- Windows to be glued & riveted, but riveted only
- Square windows → oval
- Skin thickened
- Service in 1958
I-35 bridge failure

By Laurie Blake, Paul Mcenroe, Pat Doyle and Tony Kennedy, Star Tribune

MnDOT’s options:

- Make repairs or find flaws & bolt on steel plating
- Fueled emotional debate

MnDOT’s action:

- Thousands of bolt holes would weaken bridge
- Launched inspection, interrupted by work on bridge surface

The state's top bridge engineer:

- "We chose the inspection route….. We thought we had done all we could, but obviously something went terribly wrong."
- "Up until the late 1960s, it was thought that fatigue was not a phenomenon you would see in bridges."
How much do engineers know about fatigue?

How “exact” are fatigue models?
Figure by MIT OpenCourseWare. Adapted from Fig. 6-11 in Shigley & Mischke.
Experimental data

Tensile strength $S_{tu}$, kpsi

Endurance limit $S'_e$, kpsi

$S'_e / S_{tu} = 0.6$

Figure by MIT OpenCourseWare. Adapted from Fig. 6-17 in Shigley & Mischke.
What actions and/or practices should be put in place as a result?
Testing and prevention

Where life-limb-$ are important

- It is your job to spec out test type and procedure
- Balance of cost vs. risk

Example types

- Ultrasonic
- Liquid penetrant
- Stiffness/impulse
- Eddy-current
- Leaks
- Visual

Many people listen to REAL data

Few people listen to Eqxns

A choice: Job vs. safety

On foreseeable use

- Common sense
- Legal

Images removed due to copyright restrictions.

Please see http://www.labino.com/bilder/applications/00533_RT8.jpg

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Where do cracks come from?
Potential crack origins/causes

Inherent to material
- Imperfections, e.g. castings
- Precipitates, e.g. Al 6061 T6
- Coalescing of internal dislocations
- Grain boundaries

Fabrication-related
- Tool marks
- Improper assembly, e.g. forcing (car suspension-cast materials)
- Thermally induced - Weld cracks and related HAZ problems

Use-related
- High stress areas
- Scratches
- Unintended use/damage/loading (e.g. 3 finger tight and paint lid)
Fatigue: Origin of problem

Images removed due to copyright restrictions. Please see:

http://www.metallographic.com/Images/Zn-Al.jpg

and

Fig. 4b in Henderson, Donald W., et al. "The Microstructure of Sn in Near-Eutectic Sn-Ag-Cu Alloy Solder Joints and its Role in Thermomechanical Fatigue." Journal of Materials Research 19 (June 2004): 1608-1612

or

Fatigue life review

[Graph showing fatigue life review with axes labeled Log(S) on the y-axis and Log(10^3), Log(10^6), and Log(N) on the x-axis. The graph includes points indicating low cycle and high cycle regions.]

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Fatigue life review

Ferrous materials/allow: $S_e \sim 1000000 - 10000000$

- Under ideal conditions

Non-ferrous (i.e. aluminum) generally no $S_e$...

Do we use Al in places where fatigue is important?...

- Aircraft...
- History Channel Boneyard...

Science vs. engineering...

Methods
- Stress
- Strain
- Fracture mechanics

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Fatigue and ethical responsibility

You will be criminally negligent if you do not augment calculations with TESTING for critical fatigue applications.

Life-limb-$
Real life

http://video.google.com (author?)

What reasonable hypotheses could one hold for identifying important factors?
Fatigue life modifiers

Experimental results are used to obtain modifiers

\[ S_e = \left( k_a \ k_b \ k_c \ k_d \ k_e \ k_f \right) S'_e \]

Where:

- \( k_a \) = Surface condition modification factor
- \( k_b \) = Size modification factor
- \( k_c \) = Load modification factor
- \( k_d \) = Temperature modification factor
- \( k_e \) = Reliability modification factor
- \( k_f \) = Others…

- \( S'_e \) = Rotary-beam test endurance limit
- \( S_e \) = Predicted endurance limit for your part
Endurance limit depends on many factors

For ferrous materials, the following approximations may be used for first pass design

\[
0.5 S_{ut} \quad S_{ut} \leq 200kpsi
\]

\[
S'_{e} = 100kpsi \quad S_{ut} > 200kpsi
\]

\[
700MPa \quad S_{ut} > 1400MPa
\]

This is for ideal conditions… but designs are never ideal

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Figure by MIT OpenCourseWare. Adapted from Fig. 6-8 and 6-9 in Shigley & Mischke.
Experimental data

Figure by MIT OpenCourseWare. Adapted from Fig. 6-17 in Shigley & Mischke.

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Fatigue life modifiers: Surface condition

Experimental results are used to obtain modifiers

$$k_a = a \ S_{ut}^b$$

Where:

- $a = \text{function of fabrication process}$
- $b = \text{function of fabrication process}$

- Why does finish matter?

<table>
<thead>
<tr>
<th>Surface finish</th>
<th>Factor a</th>
<th>Exponent b</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{ut}$ ksi</td>
<td>$S_{ut}$ MPa</td>
</tr>
<tr>
<td>Ground</td>
<td>1.34</td>
<td>1.58</td>
</tr>
<tr>
<td>Machined or cold-drawn</td>
<td>2.70</td>
<td>4.51</td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>14.4</td>
<td>57.7</td>
</tr>
<tr>
<td>As-forged</td>
<td>39.9</td>
<td>272.</td>
</tr>
</tbody>
</table>
Why would surface condition matter?
### Surface roughness review

**Common surface roughness** (Ra in micro-inches)

<table>
<thead>
<tr>
<th>Process</th>
<th>2000</th>
<th>1000</th>
<th>500</th>
<th>250</th>
<th>125</th>
<th>63</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>1/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sawing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Drilling</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Milling</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grinding</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polishing</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Only specify what you need & know your processes
Why would part size matter?
Fatigue life modifiers: Size factor

For bending and torsion of a round bar:

\[ k_b = 0.879 \, d^{-0.107} \quad \text{for} \quad 0.11\text{in} < d < 2.00\text{in} \]
\[ k_b = 0.910 \, d^{-0.157} \quad \text{for} \quad 2.00\text{in} < d < 10.0\text{in} \]

For axial loading:

\[ k_b = 1 \]

What if the bar is not round?

- Use a 95 percent stress area
- Equate volumes, length drops out
- Relate cross sectional area of round and square bar

\[ d_e = 0.808 (h \, b)^{0.5} \]
Why would the type of loading matter?
Fatigue life modifiers: Loading factor

For bending and torsion of a round bar:

\[ k_c = 0.85 \]

1.00 \textit{bending}

0.59 \textit{axial}

0.59 \textit{torsion}
Why would temperature matter?
Fatigue life modifiers: Temperature factor

The effect of increasing temperature
- Yield strength typically decreases
- May be no fatigue limit for material-temperature combos

The temperature factor
- May be ESTIMATED from existing tables
- Should ALWAYS BE DETERMINED EXPERIMENTALLY FOR YOUR GIVEN MATERIAL.

Relate strength at temperature to room temp. strength

\[ k_d = \frac{S_T}{S_{RT}} \]
Fatigue life modifiers: For an example steel

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>S_T/S_RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.000</td>
</tr>
<tr>
<td>50</td>
<td>1.010</td>
</tr>
<tr>
<td>100</td>
<td>1.020</td>
</tr>
<tr>
<td>150</td>
<td>1.025</td>
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<tr>
<td>200</td>
<td>1.020</td>
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<tr>
<td>250</td>
<td>1.000</td>
</tr>
<tr>
<td>300</td>
<td>0.975</td>
</tr>
<tr>
<td>350</td>
<td>0.943</td>
</tr>
<tr>
<td>400</td>
<td>0.900</td>
</tr>
<tr>
<td>450</td>
<td>0.843</td>
</tr>
<tr>
<td>500</td>
<td>0.768</td>
</tr>
<tr>
<td>550</td>
<td>0.672</td>
</tr>
<tr>
<td>600</td>
<td>0.549</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature, °F</th>
<th>S_T/S_RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1.000</td>
</tr>
<tr>
<td>100</td>
<td>1.008</td>
</tr>
<tr>
<td>200</td>
<td>1.020</td>
</tr>
<tr>
<td>300</td>
<td>1.024</td>
</tr>
<tr>
<td>400</td>
<td>1.018</td>
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<tr>
<td>500</td>
<td>0.995</td>
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<td>600</td>
<td>0.963</td>
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<td>700</td>
<td>0.927</td>
</tr>
<tr>
<td>800</td>
<td>0.872</td>
</tr>
<tr>
<td>900</td>
<td>0.797</td>
</tr>
<tr>
<td>1000</td>
<td>0.698</td>
</tr>
<tr>
<td>1100</td>
<td>0.567</td>
</tr>
</tbody>
</table>

Figure by MIT OpenCourseWare. Adapted from Table 6-4 in Shigley & Mischke.
Part II

Calculations
How do statistics and probability come into play?
Standard normal distribution, mean = 0

Standard normal distribution curve generated via the probability distribution

- Area under the curve = 1

\[
f(z) = \frac{1}{\sqrt{2\pi} \hat{\sigma}_x} \exp \left[ -\frac{1}{2} (z)^2 \right]
\]

What if mean is not 0?

\[
f(x) = \frac{1}{\sqrt{2\pi} \hat{\sigma}_x} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu_x}{\hat{\sigma}_x} \right)^2 \right]
\]

This will be covered in the 2nd design lab
Non-zero means in Gaussian distributions

A normal Gaussian distribution is typically observed in fatigue behavior of parts

- $x = \text{variate} = x$
- $z = \text{transformation variate}$
- $\sigma = \text{standard deviation}$

$$f(x) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu_x}{\hat{\sigma}_x} \right)^2 \right]$$

$$z = \frac{x - \mu_x}{\hat{\sigma}_x}$$

$$f(z) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp \left[ -\frac{1}{2} (z)^2 \right]$$

Only one table for values needed to find the area between $z$ values…
Fatigue life modifiers: Reliability factor

\[ Z = \frac{X - \mu_x}{\hat{\sigma}_x} \]

Normal distribution curve

Standard normal distribution curve
Example use of standard normal distribution

In a shipment of 250 connecting rods, the mean tensile strength is 45 kpsi and the standard deviation is 5 kpsi

- (a) Assuming a normal distribution, how many rods may be expected to have a strength less than 39.5 kpsi?
- (b) How many are expected to have a strength between 39.5kpsi and 59.5kpsi?

$$z_{39.5} = \frac{x - \mu_x}{\sigma_x} = \frac{39.5 - 45.0}{5.0} = -1.10$$

$$\Phi(z_{39.5}) = \Phi(-1.10) = 0.1357$$

$$N \cdot \Phi(z_{39.5}) = 250 \times 0.1357 = 33.9 \approx 34$$
Fatigue life modifiers: Reliability factor

Most strength data is reported as mean values

- Standard deviations typically less than 8%, but you MUST KNOW what it is… run experiments…
- 68% of all measurements fall within one standard deviation
- 95% of all measurements fall within two standard deviations

- For $\sigma \sim 8\%$

$$k_e = 1 - 0.08 \ z_a$$
How do we do 1st order fatigue modeling/analysis?
Fluctuating stresses

Stress values of concern

- $\sigma_{\text{min}}$: Minimum stress
- $\sigma_{\text{max}}$: Maximum stress
- $\sigma_a$: Amplitude component $= (\sigma_{\text{max}} - \sigma_{\text{min}})/2$
- $\sigma_m$: Midrange component $= (\sigma_{\text{max}} + \sigma_{\text{min}})/2$
- $\sigma_s$: Steady component

- $R$: Stress ratio $= \sigma_{\text{min}} / \sigma_{\text{max}}$
- $A$: Amplitude ratio $= \sigma_a / \sigma_m$

Note the correction to $\sigma_a$ and $\sigma_m$
Fluctuating stresses

Figure by MIT OpenCourseWare. Adapted from Fig. 6-25 in Shigley & Mischke.
Fatigue diagram: Goodman

Capital $S = \text{strength!}$

$$r = \frac{S_a}{S_m} = \frac{\sigma_a}{\sigma_m}$$

$$\frac{S_y}{S_e} + \frac{S_x}{S_{ut}} = 1$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Load line

Modified Goodman line
Stress concentration and notch sensitivity

Fatigue is due to crack propagation, hence notch sensitivity is important

- Max stress

\[ \sigma_{\text{max}} = K_f \sigma_o \]

\[ \tau_{\text{max}} = K_{fs} \tau_o \]

- Notch sensitivity, q (usually 0 < q < 1) accounts for material sensitivity

  - \[ K_f = 1 + q (K_t - 1) \]
  - \[ K_{fs} = 1 + q_{\text{shear}} (K_{ts} - 1) \]

It is always safe to use \( K_t \)

\( K_t \) rarely > 3 for good/practical designs, but check!
Example

1.5 in diameter AISI 1050 cold drawn steel ($S_y = 84\text{kpsi}$, $S_{ut} = 100\ \text{kpsi}$) withstands a tensile load that ranges from 0 to 16000 lbf. $K_f = 1.85$, $k_a = 0.797$, $k_b = 1$, $k_d = 1$, $k_c = 0.923$. (8th edition has $k_c = 0.85$)

Modifications in example:

- $k_c = 0.85$ in 8th edition  
  Example modified for 8th edition
- $Se = \frac{1}{2} S_{ut}$ in 8th edition

- a. Factor of safety if $\sigma_a$ held constant
- b. Factor of safety if $\sigma_m$ held constant
- c. Factor of safety if $\sigma_a/\sigma_m = \text{constant}$

$$Se = k_a k_b k_c k_d S'_{e} = k_a k_b k_c k_d (0.5\ S_{ut}) = 33.9\text{kpsi}$$
Part b: $\sigma_a$ held constant

Before applying $K_f$

$\sigma = 4.5 \text{kpsi}$

After applying $K_f$

$\sigma = 8.38 \text{kpsi}$

\[ n = \frac{S_a}{\sigma_a} = \frac{75.3 \text{kpsi}}{8.38 \text{kpsi}} = 8.98 \]
Part a: $\sigma_m$ held constant

$$n = \frac{S_a}{\sigma_a} = \frac{31.1 \text{kpsi}}{8.38 \text{kpsi}} = 3.71$$

$\sigma_a$ [kpsi]

$\sigma_m$ [kpsi]
Part c: $\sigma_a / \sigma_m$ held constant

\[ n = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{25.3\text{kpsi}}{8.38\text{kpsi}} = 3 \]
What about your shaft?

Step 1: Free body diagram
- Cutting forces (2.008 person and/or next week)
- Driving loads
- Reaction loads
- Preloads
- Others… OS! loads?

I can be here Saturday to help, if people ask!!!

Step 2: Parametric geometry & load variables

Step 3: Material properties

Step 4: Force magnitudes estimates/calculations

Step 5: Stress & fatigue
- \( \sigma_x, \sigma_y, \tau_{xy}, K_t, q, K_f, \sigma_a, \sigma_m, \sigma_s, \sigma_x \)

In the end, SH… so you should program this into excel:
- In case you need to change variables… there are always changes!
- Optimization in excel.