Reading and plans

Shigley-Mischke sections
- None

Today: Actuators: Hydraulic and Electromagnetic
- Energy transfer and scale
- Hydraulic / fluidic
- DC Permanent magnet motors
- Perhaps…. wrap up of MEMS
Friction-based machines

**Purpose:**
- Do work at a given rate, Energy - Power
- Physics: Energy and mass conservation/balances

**Characteristics of import**
- Load
- Speed
- Bandwidth
- Cost
And there can be other issues of import…

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Consequences


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An unpleasant (I hope) example
Common actuators for mechanical systems

- Biological
- Pneumatic/Hydraulic
- Electromagnetic
- Electrostatic
- Piezo
- Thermal
Biological
People powered machines

Energy

Power

Load

Speed

Bandwidth

Why is it important to understand what humans can do?
Hydraulics
Basic principles
Sub-system design

Pump
- \( T_p = \) ?
- \( D_p = \frac{1}{2} \text{ in}^3/\text{rev} \)
- \( \omega_p = 100 \text{ rpm} \)

Motor
- \( D_m = 2 \text{ in}^3/\text{rev} \)
- \( \omega_2 = 100 \text{ rpm} \)
- \( N_m = 40 \)

Output shaft
- \( T_2 = \) ?

Apiston = 1 in²

\( N_2 = \) ?

\( \omega_2 = 100 \text{ rpm} \)

Flow direction
Examples: Real but practical ;) ?

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http://gizmodo.com/
Other less than practical examples

Other less than practical examples

Please see any video of a hydraulic low rider assembly.
Other less than practical examples

Hydraulic systems in machines

Advantage:
- High force/torque and routing of power

Disadvantage:
- Leaking and wear due to contaminants

Liquids & gases in fluid-based machinery
- Hydraulics: Fluid is a liquid
- Pneumatics: Fluid is a gas
Example: Piston pump doing work

Hydraulic machines can be used to do work

- Load on the system extracts energy from the liquid
- Pressure increases between the input and output components
- Pressure is used to do work on internal parts of hydraulic devices
- Power is input/extracted via shaft (motor) or rod (cylinder)
Volume flow rate and displacement

Displacement (D)
- Displacement = volume of fluid moved / cycle
- Cycle = rotation (drill pump) or stroke (cylinder)

\[ Q = \text{volume moved per unit time} \]

\[ D \times f = Q \]

\[ \downarrow \quad \downarrow \quad \downarrow \]

\[ \text{Volume} \times \text{Cycles} = \text{Volume} \]

\[ \text{Cycle(s)} \quad \text{second} \quad \text{second} \]

- \( F \) is the frequency of a machine’s cycle
  - For hydraulic pumps, \( f = \text{speed of the shaft} = \omega/(2\pi) \) \( \omega \) [rad/s]
  - For hydraulic motors, \( f = \text{speed of the shaft} = \omega/(2\pi) \) \( \omega \) [rad/s]
  - For cylinders, \( f = \text{strokes/second} \) \( f \) [Hertz]
Displacement: Physical example

D = volume pumped per cycle
1 Cycle = expansion + contraction

\[ V_{initial} = \frac{4}{3} \cdot \pi \cdot (r_{initial})^3 \]

\[ V_{final} = \frac{4}{3} \cdot \pi \cdot (r_{final})^3 \]

\[ D = V_{initial} - V_{final} = \frac{4}{3} \cdot \pi \cdot [\left(r_{initial}\right)^3 - \left(r_{final}\right)^3] \]
Incompressibility

Incompressible fluid:

Compressible fluid:
Why is incompressibility important?

Mass balances

- The mass density ($\rho_m$) of fluids changes with pressure ($\Delta p$)

- Compressible fluids: exhibit large ($\Delta \rho_m$) for small ($\Delta p$)

- If ($\Delta \rho_m$) is large, it is possible to store significant mass in a machine

- This complicates our analysis

\[
\Sigma \dot{m}_{in} = \Sigma \dot{m}_{out} + \frac{d}{dt} m_{stored}
\]
Energy balances

- All fluids store energy when compressed (similar to a spring)
- Compressible fluids store A LOT of energy (think balloons!!)
- Stored energy complicates analysis (calculating can be difficult)

\[ \Sigma E_{in} = \left[ \Sigma E_{out} \right] + \Sigma E_{stored} \]
Example: “Locked” piston positions

Incompressible fluid:

Compressible fluid:
Incompressibility

Bulk modulus: Measures of resistance to Δvolume

\[ \beta = - \frac{dp}{\left( \frac{dV}{V_{\text{initial}}} \right)} \]

For small (incremental) changes in volume: \( \beta \approx - \frac{\Delta p}{\left( \frac{\Delta V}{V_{\text{initial}}} \right)} \)

Example: Fluid in tube exposed to pressure increase
Incompressibility

Hydraulics, pneumatics and incompressibility

- Pneumatics = gas: Low $\beta$, usually compressible
- Hydraulics = liquid: High $\beta$, usually incompressible

What makes a good assumption?

- Depends on the error you are willing to live with

Example: Incompressibility of water (e.g. H$_2$O)

$\beta_{H_2O} = 2.2 \times 10^9 \text{ N/m}^2 = 3.2 \times 10^5 \text{ lbf/in}^2 \implies \left( \frac{\Delta V}{V} \right) \approx - \frac{\Delta p}{\beta}$

Example for H$_2$O where $\Delta p = 2500 \text{ psi}$, $\frac{\Delta V}{V} = -0.006 = -0.6\%$

Is this OK?
Volume flow rate, \( Q \)

**Link between mass flow rate & volume flow rate:**

\[ Q = \text{time rate of volume flow through a hydraulic system} \]

**From mass conservation**

\[
Q_i = \frac{\dot{m}_i}{\rho_{mi}} = \rho_{mi} \cdot A_i \cdot v_i
\]

\[
\frac{\Sigma \dot{m}_{in}}{\rho_{min}} = \frac{\Sigma \dot{m}_{out}}{\rho_{mout}} + \frac{d}{dt} \frac{m_{stored}}{\rho_{mstored}} \rightarrow Q_{in} = Q_{out} + \frac{d}{dt} \left( V_{stored} \right)
\]

Mass densities are equal and cancel out of equation if fluid is incompressible

For incompressible flow in a pipe:

\[ A_{in} \cdot v_{in} = Q_{in} = Q_{out} = A_{out} \cdot v_{out} \]
Vane pumps

Series of vanes extending radially from rotating core

- Vanes can slide in/out or deform depending upon design

How a sliding vane pump works:

- Step 1: Fluid enters when volume between vanes is increasing
- Step 2: Fluid travels when volume between vanes does not change
- Step 3: Fluid exits at when volume between vanes is decreasing

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Pump types: Piston

How it works:

- Step 1: Piston forces fluid out during initial stroke
- Step 2: Valves change fluid path (only allows flow into pump)
- Step 3: Piston recharged with fluid, cycle starts again

What is the displacement?
Pump types: Piston

How it works:

- Step 1: Piston forces fluid out during initial stroke
- Step 2: Valves change fluid path (only allows flow into pump)
- Step 3: Piston recharged with fluid, cycle starts again

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http://www.animatedsoftware.com/pics/pumps/wobble.gif
http://www.flexicad.com/bilder/Rhino_Galerie/Kolpenpumpe.jpg
Pump types: External gear pump

Only one gear is driven, the other spins free

Which way does the flow go?

- Step 1: Fluid comes in at ?
- Step 2: Fluid travels through ?
- Step 3: Fluid exits at ?

What is the displacement?
Pump types: External gear pump

a) Pump Body

a) Gear and Shaft
Hydraulics Exercise
Competition: Pump

Form group

- In 10 minutes, make best estimate of gear pump displacement
- Hand in answer/analysis at end of exercise (with all names)
- Sketches, calculations, etc… must be handed in before bell sounds

Gear Pump Cavity Plate with Dimensions
All Dimensions in mm

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Hydraulics power
Power example: Pump at steady state

\[ A_{in} \cdot v_{in} = Q_{in} = Q_{out} = A_{out} \cdot v_{out} \]
Example: Pump at steady state

Pressure force does work on fluid entering pump

Work done on fluid via shaft input

Pump does work on Fluid exiting pump

Pump

$T_{\text{shaft}}$

$A_{\text{in}}$

$A_{\text{out}}$

$P_{\text{in}}$

$P_{\text{out}}$

$V_{\text{in}}$

$V_{\text{out}}$
Example: Pump at steady state

\[ \Sigma P_{in} = \left[ \Sigma P_{out} \right] + \frac{d}{dt} \left( E_{stored} \right) \rightarrow P_{inlet} + P_{shaft} = \left[ P_{outlet} + P_{loss} \right] + \frac{d}{dt} \left( E_{stored} \right) \]

\[ P_{inlet} = \left[ F_{in} \right] \cdot v_{in} = \left[ p_{in} \cdot A_{in} \right] \cdot v_{in} \]

\[ P_{shaft} = T_{shaft} \cdot \omega_{shaft} \]

\[ P_{outlet} = \left[ F_{out} \right] \cdot v_{out} = \left[ p_{out} \cdot A_{out} \right] \cdot v_{out} \]

If can assume A&B, \( P_{loss} \) & \( \frac{d(E)}{dt} \) can be neglected

A. \( \frac{d}{dt} \left( E_{stored} \right) \ll\ll P_{in} - P_{out} \)

B. \( P_{loss} \ll\ll P_{in} - P_{out} \)

Substituting into energy balance (top equation on sheet)

\[ \left[ p_{in} \cdot A_{in} \right] \cdot v_{in} + T_{shaft} \cdot \omega_{shaft} = \left[ \left[ p_{out} \cdot A_{out} \right] \cdot v_{out} + \sim 0 \right] + \sim 0 \]
Power example: Pump at steady state

\[ [P_{in} \cdot A_{in}] \cdot v_{in} + T_{shaft} \cdot \omega_{shaft} = \left[ [P_{out} \cdot A_{out}] \cdot v_{out} + \sim 0 \right] + \sim 0 \]

\[ A_{in} \cdot v_{in} = Q_{in} = Q_{out} = A_{out} \cdot v_{out} \]

\[ Q_{in} = Q_{out} = Q \]

\[ [P_{out} - P_{in}] \cdot Q = T_{shaft} \cdot \omega_{shaft} \]
Hydraulics
System example
Power example: Pump at steady state

$D_p = 0.5 \text{ in}^3/\text{rev}$
$T_p = 10 \text{ in-lbf}$
$\omega_p = \frac{1000 \text{ rpm}}{2\pi}$

$D_m = 0.5 \text{ in}^3/\text{rev}$
$T_m = ?$
$\omega_m = ?$

$p_1 = 10 \text{ psi}$
$p_2 = ? \text{ psi}$
$p_3 = 10 \text{ psi}$

Density = $\rho$
DC permanent magnet motors
**DC Permanent magnet motor**

**T vs. ω for Black & Decker Screw Driver**

\[ T \text{ [N-m]} = -0.012 \text{ [N-m/rpm]} \times \omega + 3.678 \text{ [N-m]} \]

<table>
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<tr>
<th>ω [rpm]</th>
<th>Data T [N-m]</th>
<th>Fitted T [N-m]</th>
<th>% Error</th>
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<td>310</td>
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<td>-0.09</td>
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</table>
Understanding the model

Simple 1 loop model
- Goal: understand trends

Assumptions
- Low loss in wires
- Steady state
- Single loop
- No ferrous cores
- Snap shot with loop plan in the y-z plane
Point we will study

Points we are studying

Torque curve of simple loop

Side view of simple loop
Forces

Force on wire

\[ \vec{F}_B = i \cdot (\vec{L} \times \vec{B}) \]

L points in direction of current flow

MAGNETIC FLUX DENSITY, B

CURRENT, \( i_{\text{wire}} \)

MAGNETIC FORCE

Lorentz Force

\[ \vec{F}_E = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B} \]
Torque inducing forces on wire

**Force on wire**

\[ \vec{F}_B = i \cdot (\vec{L} \times \vec{B}) \]

**Torque at \( \omega = 0 \)**

\[ \vec{T} = 2 \cdot (\vec{r} \times \vec{F}_B) \]

\[ |T| = 2 \cdot r \cdot (i \cdot L \cdot B) \cdot \sin(\theta_{r-F}) \]

\[ T = 2 \cdot r \cdot \frac{(V_1 - V_4)}{R} \cdot L \cdot B \]

\[ (V_1 - V_4)_{Battery} = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} \]
Lorentz force

Force due to \( E \& B \)

\[
\vec{F}_E = q \cdot \vec{E} + q \cdot (\vec{v} \times \vec{B})
\]

\[
|E| = |v| \cdot |B| \cdot \sin(\theta_{v-B})
\]

Wire 1-2

\[
\frac{(V_2 - V_1)}{\omega} = (r \cdot \omega) \cdot B
\]

Wire 2-3

\( \vec{v} \times \vec{B} \) not along \( r \)

\[
V_2|_\omega = V_3|_\omega
\]

Wire 3-4

\[
\frac{(V_4 - V_3)}{\omega} = (r \cdot \omega) \cdot B
\]
Induced voltage due to rotation

\[ \Delta V \text{ due to rotation, } \omega \]

\[ \frac{(V_2 - V_1)}{L} = (r \cdot \omega) \cdot B \]

\[ V_2|_\omega = V_3|_\omega \]

\[ \frac{(V_4 - V_3)}{L} = (r \cdot \omega) \cdot B \]

\[ \frac{(V_4 - V_1)}{L} = 2 \cdot (r \cdot \omega) \cdot B \]

\[ (V_1 - V_4)|_\omega = -2 \cdot (r \cdot \omega) \cdot B \cdot L \]
Total voltage

\[ \Delta V \text{ due to rotation, } \omega \]

\[ (V_1 - V_4)_{\omega} = -2 \cdot (r \cdot \omega) \cdot B \cdot L \]

\[ \Delta V \text{ due to battery} \]

\[ (V_1 - V_4)_{\text{Battery}} = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} \]

Total potential diff.

\[ \Delta V = (V_1 - V_4)_{\text{Battery}} + (V_1 - V_4)_{\omega} \]

\[ \Delta V = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} - 2 \cdot (r \cdot \omega) \cdot B \cdot L \]
Torque – $\omega$ relationship

Total potential diff.
$$\Delta V = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} - 2 \cdot (r \cdot \omega) \cdot B \cdot L$$

Ohm’s law
$$\Delta V = i \cdot R$$

Total potential diff.
$$i \cdot R = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} - 2 \cdot (r \cdot \omega) \cdot B \cdot L$$

T- $\omega$ relationship
$$T = 2 \cdot i \cdot L \cdot r \cdot B - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R} \omega$$
Torque – $\omega$ relationship cont.

T- $\omega$ relationship

$$T = 2 \cdot i \cdot L \cdot r \cdot B - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R} \omega$$

Stall torque

$$T_{Stall} = 2 \cdot i \cdot L \cdot r \cdot B$$

T- $\omega$ relationship

$$T = T_{Stall} - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R} \omega$$
Torque – $\omega$ relationship cont.

$$T = T_{\text{Stall}} - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R} \omega$$

### T- $\omega$ curve

**T vs. $\omega$ for Black & Decker Screw Driver**

$$T_{\text{Stall}} = -0.012 \text{ [N-m/rpm]} \cdot \omega + 3.678 \text{ [N-m]}$$
Scaling
Follow up on micro-actuator lecture
Electrostatics

How does electrostatic physics scale?

\[ U_E = \frac{\varepsilon_o \cdot L \cdot L \cdot V^2}{2 \cdot z} \]

How does ratio of \( F_{\text{Electric}} \) scale to \( F_{\text{Body}} \)?

\[
\left| \frac{F_{\text{Electric}}}{F_{\text{Body}}} \right| \sim \frac{1}{L}
\]

What does this mean for MuSS interaction?

- What happens if you downsize each by factor of 10?
Electrostatics

\[ U_{Electric-z} = \frac{\varepsilon_o \cdot L \cdot L \cdot V^2}{2 \cdot z} \quad \Rightarrow \quad F_{Electric-z} = -\frac{dU}{dz} \quad \Rightarrow \quad F_{Electric-z} = \frac{\varepsilon_o \cdot L^2 \cdot V^2}{2 \cdot z^2} \]

\[ F_{body} = \rho \cdot V^3 \quad \Rightarrow \quad \frac{F_{Electric}}{F_{Body}} \sim \frac{1}{L} \]

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Semi-intuitive example

Cooling...

Heating…
Thermal behavior

How does thermal physics scale (small Bi #)?

$$e^{-\left(\frac{h \cdot A}{\rho \cdot V \cdot c}\right) \cdot t} = \frac{\theta}{\theta_{\text{inf}}} = \frac{T - T_{\text{inf}}}{T_{\text{initial}} - T_{\text{inf}}}$$

$$Bi = \frac{h \cdot L}{k} \sim \text{Convection}$$

Conduction
Thermal behavior

How does thermal physics scale?

\[-h \cdot A \cdot (T - T_{\text{inf}}) = \rho \cdot c \cdot V \cdot \frac{dT}{dt}\]

\[e^{-\left(\frac{h \cdot A}{\rho \cdot V \cdot c}\right) t} = \frac{\theta}{\theta_{\text{inf}}} = \frac{T - T_{\text{inf}}}{T_{\text{initial}} - T_{\text{inf}}}\]

\[\tau \sim \frac{\rho \cdot V \cdot c}{h \cdot A} \rightarrow L\]

Is this a good or a bad thing for MEMS actuators?

For the STM?
Fluidics

How do fluid-based physical phenomena scale?

\[ Q = \frac{\pi r^4 \Delta p}{8 \cdot \mu \cdot L} \]

\[ Q = U \cdot \pi \cdot r^2 \]

\[ \Delta p = -\frac{8 \cdot \mu \cdot U}{r^2} \cdot L \]

High pressure change over narrow flow paths…
Fluidics

Reynolds number

\[ \text{Re} = \frac{\rho \cdot U \cdot D}{\mu} \]

Ratio of inertial forces to viscous forces

\[ D = 50 \, \mu m \quad U = 500 \, \mu m/s \quad L = 1000 \, \mu m \]

\( \text{Re}_{\text{Air}} \) and \( \text{Re}_{\text{H2O}} \) << 1

What does this mean:

- Heavily damped
- Limits response time (ms vs. \( \mu s \))