MRP Overview

• **Primary source:** *Factory Physics* by Hopp and Spearman.

• **Basic idea:** Once the final due date for a product is known, and the time required for each production step is known, then intermediate due dates and material requirement times can be determined.

• **Original goal:** To determine when material for production is required.
Demand

- ... from outside the system is *independent* demand.
- ... for components or raw material is *dependent* demand.

Before MRP, buyers were not synchronized with producers.
MRP Overview

- Start at the due date for a finished product (or end item) \( (T_k) \).
- Determine the last operation, the time required for that operation \( (t_{k-1}) \), and the material required for that operation.
- The material may come from outside, or from earlier operations inside the factory.
- Subtract the last operation time from the due date to determine when the last operation should start.
MRP Overview

Planning Algorithm

\[ T_{k-1} = T_k - t_{k-1} \]

- The material required must be present at that time.
- Continue working backwards.
- However, since more than one component may be needed at an operation, the planning algorithm must work its way backwards along each branch of a tree — the bill of materials.
In some MRP systems, time is divided into *time buckets* — days, weeks, or whatever is convenient.

In others, time may be chosen as a continuous variable.
MRP Overview

Discussion

- What assumptions are being made here ...
  - ... about predictability?
  - ... about capacity?
- How realistic are those assumptions?
- Is it more flexible to use time buckets or continuous time?
MRP Overview

- **Push system:** one in which material is loaded based on planning or forecasts, not on current demand.
  - MRP is a push system.

- **Pull system:** one in which production occurs in response to the consumption of finished goods inventory by demand.

- Which is better?
Bill of Materials (BOM)

- Top level is end item.
- Items are given a low-level code corresponding to the lowest level they appear at, for any end item in the factory.

The BOM must be maintained as the product mix changes.
Master Production Schedule

- Information concerning independent demand.
- *Gross requirements*: What must be delivered in the future, and when.
- *On-hand inventory*: Finished good already available.
- *Net requirements*: (Gross requirements) – (On-hand inventory).
### Master Production Schedule

#### Example Netting

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirements</td>
<td>15</td>
<td>20</td>
<td>50</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Projected on-hand</td>
<td>30</td>
<td>15</td>
<td>-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net requirements</td>
<td>0</td>
<td>5</td>
<td>50</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

- 15 of the initial 30 units of inventory are used to satisfy Week 1 demand.
- The remaining 15 units are 5 less than required to satisfy Week 2 demand.
Lot sizes are 75. The first arrival must occur in Week 2.
75 units last until Week 4, so plan arrival in Week 5.
Similarly, deliveries needed in Weeks 5 and 7.

<table>
<thead>
<tr>
<th></th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirements</td>
<td>15</td>
<td>20</td>
<td>50</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Cumulative gross</td>
<td>15</td>
<td>35</td>
<td>85</td>
<td>95</td>
<td>125</td>
<td>155</td>
<td>185</td>
<td>215</td>
</tr>
<tr>
<td>Planned order receipts</td>
<td>30</td>
<td>0</td>
<td>75</td>
<td>0</td>
<td>0</td>
<td>75</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>Cumulative receipts</td>
<td>30</td>
<td>105</td>
<td>105</td>
<td>105</td>
<td>180</td>
<td>180</td>
<td>255</td>
<td>255</td>
</tr>
</tbody>
</table>
Material requirements are determined by considering whether inventory would otherwise become negative.
• Lead times are 1 week. Therefore, order *release* must occur one week before delivery is required.

<table>
<thead>
<tr>
<th></th>
<th>Week</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3 4 5 6 7 8</td>
</tr>
<tr>
<td>Gross requirements</td>
<td>15  20 50 10 30 30 30 30</td>
</tr>
<tr>
<td>Cumulative gross</td>
<td>15  35 85 95 125 155 185 215</td>
</tr>
<tr>
<td>Planned receipts</td>
<td>30  0 75 0 0 75 0 75 0</td>
</tr>
<tr>
<td>Cumulative receipts</td>
<td>30 105 105 105 180 180 255 255</td>
</tr>
<tr>
<td>Planned release</td>
<td>75 0 0 75 0 75 0 0 0</td>
</tr>
</tbody>
</table>
• Now, do exactly the same thing for all the components required to produce the finished goods (level 1).

• Do it again for all the components of those components (level 2).

• Et cetera.
Data

- *Master Production Schedule*  — demand – quantities and due dates
- *Item Master File*  — for each part number: description, BOM, lot-sizing, planning lead times
- *Inventory Status*  — finished goods, work-in-progress
Data

- Planned order releases
- Change notices
- Exception reports — discrepancies, consequences of unexpected events
• Define

★ \( D_t \) = Demands, or gross requirements for week \( t \)
★ \( S_t \) = Quantity that will be completed in week \( t \)
★ \( I_t \) = Projected finished inventory in week \( t \)
★ \( N_t \) = Net requirements in week \( t \)
Inventory dynamics: Starting with $t = 1$ (where $t = 0$ means now) and incrementing $t$ by 1,

$$I_t = I_{t-1} - D_t,$$

as long as $I_{t-1} - D_t \geq 0$

$$I_t = I_{t-1} - D_t + S_t,$$

if $I_{t-1} - D_t < 0$

where $S_t$ is minimal amount needed to make the inventory positive, which is consistent with lot-sizing requirements.
Net requirements: Let \( t^* \) be the first time when \( I_{t-1} - D_t < 0 \). Then,

\[
N_t = \begin{cases} 
0 & \text{if } t < t^* \\
I_{t-1} - D_t < 0 & \text{if } t = t^* \\
D_t & \text{if } t > t^* 
\end{cases}
\]

- From net requirements, we calculate required production (scheduled receipts) \( S_t, t > t^* \).
- \( S_t \) is adjusted for new orders or new forecasts.
- Then procedure is repeated for the next \( T^* \).
## Master Production Schedule

### Netting Example

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<tr>
<th></th>
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<td>50</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Projected on-hand</td>
<td>20</td>
<td>5</td>
<td>5</td>
<td>55</td>
<td>45</td>
<td>15</td>
<td>-15</td>
<td></td>
</tr>
<tr>
<td>Adjusted scheduled receipts</td>
<td>20</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net requirements</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>30</td>
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</table>
## Master Production Schedule

### Netting Example

|                     | Week
<table>
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</tr>
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<td>Projected on-hand</td>
<td>20</td>
</tr>
<tr>
<td>Net requirements</td>
<td>0</td>
</tr>
<tr>
<td>Scheduled receipts*</td>
<td>10</td>
</tr>
<tr>
<td>Adjusted scheduled receipts</td>
<td>0</td>
</tr>
</tbody>
</table>

*original plan*

- The 10 units planned for week 1 were *deferred* to week 2.
- The 100 units planned for week 4 were *expedited* to week 3.
Possible rules:

- **Lot-for-lot**: produce in a period the net requirements for that period. *Maximum* setups.

- **Fixed order period**: produce in a period the net requirements for $P$ periods.

- **Fixed order quantity**: always produce the same quantity, whenever anything is produced. EOQ formula can be used to determine lot size.

*Which satisfy the Wagner-Whitin property?*
Lot Sizing

Master Production Schedule

cumulative gross ——
cumulative receipts —— fixed batch
Cumulative receipts —— fixed period

week

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• After scheduling production for all the top level (Level 0) items, do the same for Level 1 items.

• *The planned order releases for Level 0 are the gross requirements for Level 1.*

• Do the same for Level 2, 3, etc.
• MRP is deterministic but reality is not. Therefore, the system needs *safety stock* and *safety lead times*.
• Safety stock protects against quantity uncertainties.
  ★ You don’t know how much you will make, so plan to make a little extra.
• Safety lead time protects against timing uncertainties.
  ★ You don’t know exactly when you will make it, so plan to make it a little early.
Instead of having a minimal planned inventory of zero, the (positive) safety stock is the planned minimal inventory level.

Whenever the actual minimal inventory differs from the safety stock, adjust planned order releases accordingly.
Reality

Uncertainty/Variability

Safety Lead Time

- Add some extra time to the planned lead time.
• **Yield** = expected fraction of parts started that are successfully produced.
• Actual yield is random.
• If you need to end up with \( N \) items, and the yield is \( y \), start with \( N/y \).
• However, the actual production may differ from \( N \), so safety stock is needed.
Reality

• Capacity is actually finite.
• Planned lead times tend to be long (to compensate for variability).

★ Also, workers *should* start work on a job as soon as it is released, and relax later (usually possible because of safety lead time). Often, however, they relax first, so if a disruption occurs, the job is late.
• *Nervousness* — drastic changes in schedules due to small deviations from plans — *(chaos?*)

• Nervousness results when a deterministic calculation is applied to a random system, and local perturbations lead to global changes.
Reality

Other problems

Nervousness

cumulative gross

cumulative receipts -- original

cumulative receipts -- perturbed

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Reality

Other problems

Nervousness

● Possible consequences:
  ★ Drastic changes in plans for the near future, which will confuse workers;
  ★ Excessive setups, consuming excess expense or capacity.

● Solution: Freeze the early part of the schedule (ie, the near future). Do not change the schedule even if there is a change in requirements; or do not accept changes in requirements.

★ But: What price is paid for freezing?
Reality

- **MRP is a solution to a problem whose formulation is based on an unrealistic model**, one which is
  - deterministic
  - infinite capacity
- As a result,
  - it frequently produces non-optimal or infeasible schedules, and
  - it requires constant manual intervention to compensate for poor schedules.
- **On top of that, nervousness amplifies inevitable variability.**
MRP II

• Manufacturing Resources Planning
  ★ MRP, and correction of some its problems,
  ★ demand management,
  ★ forecasting,
  ★ capacity planning,
  ★ master production scheduling,
  ★ rough-cut capacity planning,
  ★ capacity requirements planning (CRP),
  ★ dispatching,
  ★ input-output control.
MRP II Hierarchy

Hierarchical Planning and Scheduling

Cumulative Production

Short range

Medium range

Long range

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Higher levels deal with longer time scales and more of the system (scope).

Higher levels use more aggregated (coarse-grained) models.

Higher levels send *production targets* down to lower levels.

- Each level refines the target for the level below, with reduced time scale and reduced scope.
- The bottom level actually implements the schedule.
MRP II Hierarchy

Hierarchical Planning

General ideas

Production targets → Level \( k-1 \) → Capacity reports

Production targets → Level \( k \) → Uncontrolled events

Uncontrolled events → Level \( k \) → Material movement commands

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MRP II Hierarchy

Hierarchical Planning

Long-Range Planning

- Range: six months to five years.
- Recalculation frequency: 1/month to 1/year.
- Detail: part family.
- Forecasting
- Resource planning — build a new plant?
- Aggregate planning — determines rough estimates of production, staffing, etc.
• Demand management — converts long range forecast and actual orders into detailed forecast.
• Master production scheduling
• Rough-cut capacity planning — capacity check for feasibility.
• CRP — better than rough cut, but still not perfect. Based on infinite capacity assumption.
**MRP II Hierarchy**

**Hierarchical Planning**

**Short-Term Control/Scheduling**

- **Daily Plan**
  - Production target for the day
- **Shop Floor Control**
  - Job dispatching — which job to run next?
  - Input-output control — release
  - Often based on simple rules
  - Sometimes based on large deterministic mixed (integer and continuous variable) optimization

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MRP II Hierarchy

Hierarchical Planning

Issues

• The high level and low level models sometimes don’t match.
  ★ The high level aggregation is not done accurately.
  ★ Actual events make the production target obsolete.
  ★ Consequence: Targets may be infeasible or too conservative.

• The short-term schedule may be recalculated too frequently.
  ★ Consequence: Instability.
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