Rigid Part Mating

• Goals of this class
  – understand the phases of a typical part mate
  – determine the basic scaling laws
  – understand basic physics of part mating for simple geometries
  – relate forces and motions arising from geometric errors
  – compare logic branching and direct error-feedback part mating strategies
Basic Bandwidth Issues and Time-Mass-Distance Scaling Laws

• Torque required to move a mass $M$ at the end of an arm of length $L$ in time $T$ is proportional to
  – $M \frac{L^2}{T^2}$

• This implies that really fast motions must be really small or use a small arm with small mass

• I estimated
  – my hand’s mass = 250g, effective length = 10cm
  – my arm + hand’s mass = 1700g, effective length = 35 cm
  – ratio arm:hand of $ML^2 = T^2 = 85$

• Don’t forget: arm mass+payload mass=M
Main Phases of a Part Mating Event

Approach  Chamfer Crossing  One-point Contact  Two-point Contact  Line Contact
Required Bandwidth for Chamfer Crossing

Fourier coefficient = \( 2 \pi T / (n^2 \pi^2 \tau) \sin(2n\pi\tau/T) \)

\[ T = 20 \, \text{E/V}; \quad \tau = \text{E} / 2 \, \text{V}; \quad T / \tau = 40 \]

Period = \( 2\pi = \omega T = \omega 20\text{E/V} \)

\[ \omega = \pi V / 10 \, \text{E} \]

\[ f = V / 20 \, \text{E} \]

If \( V = 10 \, \text{in/s} \) and \( E = 0.05" \), \( f = 10 \, \text{Hz} \)

If 5th harmonic must be adhered to, bandwidth needed = 50Hz
Trapezoidal Wave Harmonics

Source:
Conclusions

- Gross motions can be (must be) done by large arms that necessarily will move slowly.
- No robot arm with practical reach can make fine motion error removal adjustments at 50 Hz.
- Fine motions can be fast if they are done by small arms, and must be fast to absorb typical errors at economical speeds.
- Big tasks with big parts will take a long time compared to small tasks with small parts.
- What we see: small parts cycle times are ~5s while big parts cycle times are ~ 60s.
Essentials of Part Mating Theory for Fine Motions

- Quasi-static assumption
- Geometry of pegs and holes
- Applied forces
- Normal reaction forces and friction reaction forces
- Entry geometry limits
- Wedging conditions
- Jamming conditions
- Alternate strategies for accomplishing fine motion
The Basic Idea

• In gross motions, it pays to pre-plan to prevent errors
• In fine motion, it does not pay to try to prevent errors
• So the principle is to anticipate errors and figure out how to make assembly happen anyway
• This requires us to understand three factors:
  – Geometry
  – Compliance
  – Friction
Geometry of Peg-Hole Mates

\[ \frac{\ell}{d} = \frac{c}{\theta} \]

TYPICAL MACHINED PARTS

\[ \frac{\ell}{D} \]

PRECISION PARTS

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Source:
Dimensioning Practice

Source:
Geometry Definitions

\[ c = \frac{(D-d)}{D} \]

Insertion Direction
Insertion History
Insertion History
Insertion History
Insertion History
Insertion History
Insertion History
Insertion History
Insertion History
Life Cycle of a Part Mate

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Source:
Model of a Compliant Support

All support is assumed concentrated at one point and consists of one lateral stiffness and one angular stiffness.
How Compliance Center Reacts to Force

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Source:

Force away from C. C. causes rotation and translation

Force on C. C. causes only translation
Forces and Moments - Two Point Contact Case

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All applied and reaction forces are expressed in coordinates at peg’s tip...
Forces Applied During Two-point Contact

When $L_g >> 0$

When $L_g \sim 0$

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Making L_g Small is Good

• How to do it?
• Active Robot Force Feedback
  – Costly
  – Slow
• Some way that acts by itself
• It was invented almost 30 years ago
• Called Remote Center Compliance
• Reduces assembly force
• Avoids one of two main failure modes
CHAMFER CROSSING

ONE-POINT CONTACT

TWO-POINT CONTACT

INSERTION FORCE $F_2$, NEWTONS

INSERTION DEPTH $l$, mm

$\text{INSERTION FORCE HISTORY}$

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Assembly Failure Modes

- Both occur during two-point contact
- **Wedging** sets up compressive forces inside the parts
- **Jamming** results from incorrect insertion forces
- We can derive the requirements to avoid both of these failure modes
Wedging: Compressive Friction Forces Prevent Insertion Regardless of Insertion Force

\[ \phi = \tan^{-1} \mu \]

Wedging can happen if \( \theta > c/\mu \) when two-point contact occurs.

Wedging can be avoided if \( \mu \) is small enough or if two-point contact occurs deep enough in the hole.

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What’s a Friction Cone?

Sliding will occur if \( F_T > \mu F_N \)
\[
F_T / F_N = \tan \theta
\]
So, sliding will occur if \( \tan \theta > \mu \)
and \( F \) will lie on the boundary of the cone

If \( F \) is inside the cone then sliding will not happen
because \( F_T < \mu F_N \)
and \( F \) can be \textit{any} value
Conditions for Avoiding Wedging

\[ S = \frac{L_g}{L_g^2 + K_\theta / K_x} \]

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Source:
Jamming: Insertion Force Directed the Wrong Way - Can’t Overcome Friction

Component of Insertion force
Along insertion direction
Not big enough:
Peg Is Jammed

Component of Insertion force
Along insertion direction
Is big enough:
Peg Goes In
Conditions for Avoiding Jamming

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Source:
Jamming Examples

COMPONENT NORMAL TO PEG AXIS

COMPONENT PARALLEL TO PEG AXIS

INSERTION FORCE

FRICTION CORRESP. TO NORMAL COMPONENT

REACTION TO NORMAL COMPONENT

\[ F_x/F_z \text{ is big.} \]
\[ M/rF_z \text{ is big.} \]

\[ F_x/F_z \text{ is small.} \]
\[ M/rF_z \text{ is small.} \]
Target Expands as Depth Increases

As $\lambda$ increases

$\lambda = \frac{\ell}{2r\mu}$

This point moves

This point moves

$-(2\lambda + 1)$
Experimental Data

\[ L_g = 45 \text{ mm} \]
\[ \varepsilon_0 = 1.35 \text{ mm} \]
\[ \theta_0 = 0 \]

The diagram shows a graph with the axes labeled as follows:
- \( \ell \), mm (on the y-axis)
- \( \theta_x \), mrad (on the x-axis)

The graph includes data points labeled as "DATA" and a line labeled "THEORY."
Experimental Data -2

INSIGNER FORCE $F_z$, newtons

INSIGNER DEPTH $l$, mm

THEORY

$L_0 = 45 \text{ mm}$

$E_0 = 0.83 \text{ mm}$

$\theta_0 = 0 \text{ mm}$
Test Your Understanding

• Why does insertion force rise and then fall during two-point contact?
When $L_g = 0$ there is barely any insertion force. All that’s left is chamfer crossing force.
Test Your Understanding Again

- Why does the insertion force not rise after chamfer-crossing is finished?
Review of Force Feedback Strategy

- Create a coordinate frame at the “working point” of the part or tool
- Separate lateral and angular sensing and response motions in that frame
- Devise a response strategy
- The Remote Center Compliance is a purely passive implementation of one such strategy
Simplified Explanation of the Remote Center Compliance (RCC)

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RCC Response to External Loads

(d) RCC UNDER LATERAL LOAD

(e) RCC UNDER ANGULAR LOAD

(f) LINKAGE RCC UNDER LATERAL AND ANGULAR DEFORMATION

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(d) RCC UNDER LATERAL LOAD
(e) RCC UNDER ANGULAR LOAD
Angular Error = 3°
rigid part mating
Commercial Remote Center Compliances

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Source:
Decouples Robot Accuracy from Task Precision

Compliant Assembly Strategy
Assembly is a matter of geometry

Geometry contains errors in parts in equipment

Ignore errors (Matt Mason approach)

Accept errors

Seek perfection

Costs too much

Detect errors indirectly by sensing collisions

Detect errors directly by sensing them geometrically

But they are small and usually obscured

Collisions are bad

Collisions contain info

Collisions + stiffness matrix = force signals

Small stiffness → small signals and oscillations and position uncertainty

Large stiffness → large signals but possible instability

Active force feedback

Passive accommodation (incl small stiffness)

Can be smart can be unstable can be "strange" can be slow

Will be stable

Engineered

Accidental, contextual

With sensors → IRCC Without sensors → RCC basically Lyapunov stable

May not avoid jamming

But: friction limits the available information in force data