Math and Feature Models of Assemblies

• Start of series of 5 classes on math/CAD models
  – basic matrix representations and Feature-based Design
  – constraint
  – variation
  – assembly sequence analysis
  – Datum Flow Chain
Objectives of Assembly Modeling

• Provide a computer environment that permits top-down design of assemblies with a persistent database that captures the assembly as an assembly
• Should link to geometry creation (CAD, rule-generated)
• Should permit specification of Key Characteristics, constraints on location, datums and locators, and variation analysis for KCs using the assembly model
• Should permit assembly planning, vendor interfaces, ramp-up, and production support
Top-down and Bottom-up Design

• **Top-down** defines an assembly in this order:
  – major customer deliverables
  – chains of delivery through possible parts
  – main part mates and necessary features
  – detailed part geometry

• **Bottom-up** defines
  – the same things but in the reverse order
  – requires having some idea of final assembly layout first

• Top-down used to be the only way before CAD
• CAD seems to encourage bottom-up
Sketch of Top-Down Assembly Design

- Selected concept
- Architecture
- Integration risk
- Key dimensions

- "Location Responsibility"
- Dimensional Control
- Constraint (6 DoF)

Datum Flow Chain

Featurized DFC

Variation

Assembly Sequences

Assembly Processes

Tooling Constraints

Parts placed in DFC with features

Relating KCs to Chains

Concept Design

Assy Models
9/16/2004
© Daniel E Whitney
Goals of this Class

• Review basic math that relates adjacent coordinate frames
• Model assemblies as chains of frames
• Attach these frames to “mating features”
• Introduce feature based design for assembly
Basic Math

• Uses 4x4 matrices to relate adjacent frames
• Permits chaining together of parts
  – same math is used to describe robots
• The matrix contains a rotational part and a translational part
• The matrix is designed to translate first and then rotate so that rotation does not change position of new frame
• This matrix is a subset of a more general projection matrix that includes perspective
History of this Representation

• Basic to Kinematics (Denavit & Hartenberg)
• Used to model assemblies in 1970s:
  – S N Simunovic Master’s Thesis, MIT, 1972
  – Edinburgh University AI Lab robot assembly 1976
• Used by CAD researchers
  – Steve Coons, 1960’s
  – Gossard and others, 1980s
• Used by CAD systems to locate surfaces wrt each other
Stapler Frames and KCs

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Source:
Frames and Chains

• By following the arrows, you can travel from frame to frame
• On the previous slide, the anvil was chosen as the origin part, and the anvil-pin joint on the anvil was chosen as the location of the origin frame.
• All arrows go out from the origin frame
• You can travel from one end of a KC to the other by moving from frame to frame along the arrows, sometimes in arrow direction and sometimes in reverse
Basic Translation and Rotation

Source:

Translate first, then rotate
Basic 4x4 Transform

\[ T = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0^T & 1 \end{bmatrix} \]

\[ T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

All the information about location (position and orientation) is inside the matrix.
Basic Rotation Matrices

\[
\text{Rot}(x, \theta) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
0 & \sin \theta & \cos \theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\text{Rot}(y, \phi) = \begin{bmatrix}
\cos \phi & 0 & \sin \phi & 0 \\
0 & 1 & 0 & 0 \\
-\sin \phi & 0 & \cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\text{Rot}(z, \alpha) = \begin{bmatrix}
\cos \alpha & -\sin \alpha & 0 & 0 \\
\sin \alpha & \cos \alpha & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\text{is in radians}

No translation
Basic Translation Matrix

\[
\begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & y \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

This and the three basic rotation matrices are matlab .m files on MIT Server that you can use.
Composite Transforms

\[ T_{02} = T_{01} T_{12} \]

\[ T_{02} = \begin{bmatrix} R_{01} & p_{01} \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_{12} & p_{12} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} R_{01}R_{12} & R_{01}p_{12} + p_{01} \\ 0^T & 1 \end{bmatrix} \]

\[ \text{\( T_{01} \) locates frame 1 in frame 0 coordinates} \]
\[ \text{\( T_{12} \) locates frame 2 in frame 1 coordinates} \]
\[ \text{\( T_{02} \) locates frame 2 in frame 0 coordinates} \]
Transform Order is Important

\[ T_{AB} = T_A T_B \; . \; T_{BA} = T_B T_A \]
More About Transform Order

$T_{i+1}$ works on new axes created by $T_i$

\[ T_{1-n} = T_{01} T_{12} \cdots T_{(n-1)-n} \]
Nominal Mating of Parts

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Source:
Example

\[
T_{AB} = \begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

>> \text{trans (3,0,4)}

Coordinate Frames

MATLAB\textsuperscript{(TM)} Code
Front, Top, and Side Views

Top

Other Side

Front

Side

Bottom
Another Example

>> $T_{AC} = T_{AB} \text{ roty (dtr (90))}$

$$
T_{AC} =
\begin{bmatrix}
0 & 0 & 1 & 3 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 4 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

function degtorad = dtr(theta)
% Converts degrees to radians
degtorad=theta*pi/180;

Image removed for copyright reasons.
Source:
Figure 3-22 in [Whitney 2004] Whitney, D. E. 
Mechanical Assemblies: Their Design, Manufacture,
and Role in Product Development. New York, NY:
Example Feature on Part

>> $T_{AD} = \text{trans}(3, 2, 4) \circ \text{roty}(\text{dtr}(90))$

$$
T_{AD} = \begin{bmatrix}
0 & 0 & 1 & 3 \\
0 & 1 & 0 & 2 \\
-1 & 0 & 0 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Images removed for copyright reasons.

Source:
Figure 3-23 in [Whitney 2004] Whitney, D. E.
Note: The image is removed for copyright reasons.

Source:
Figure 3-24 in [Whitney 2004] Whitney, D. E.
Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.

\[
>> T_{EF} = trans(6,0,1)
\]
\[
T_{EF} = \begin{bmatrix}
1 & 0 & 0 & 6 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Assembling These Parts

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Source:
Figure 3-25 in [Whitney 2004] Whitney, D. E.
*Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.*

\[
>> T_{DE} = \text{rotz (dtr (180))} \\
T_{DE} = \begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
>> T_{AF} = T_{AD} T_{DE} T_{EF} \\
T_{AF} = \begin{bmatrix}
0 & 0 & 1 & 4 \\
0 & -1 & 0 & 2 \\
1 & 0 & 0 & 10 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Varied Part Location Due to Tolerances

The varied location of Part B can be calculated from the nominal location of Part A. This process can be chained to Part C, etc., including errors on Part B. It uses the same math as the nominal model.
Equations for Connective Models

Nominal

Image removed for copyright reasons.
Source:

Varied

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Source:
A Hierarchy of Assembly Models

- Bill of materials - lists the parts in no particular order
- Structured BOM - aka drawing tree - groups parts by sub-assembly
- Liaison graph - (Bourjault) parts are dots, joints are lines
- Ordered liaison graph - the lines have arrows
- Attributed liaison graph - the lines have constraint or feature information
- Ordered-attributed liaison graph (Datum Flow Chain)
Assembly Types Classified Technically

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Source:
“Assembly Types” Classified by Liaison Diagram Form

- Hub and spokes
- Loop
- Network
- Stack
Inside a Car Engine

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Source:
Feature-based Design

• Seeks to rise above geometry and capture intent
• First efforts in machined features
  – slots
  – pockets
  – holes
• Features look different depending on how they are made
  – drafted walls if cast
  – pockets if cut
• “Feature recognition” may be needed
Assembly Features

Each feature has nominal geometry and a reference coordinate frame expressed as a 4x4 matrix. It also has a variety of other attributes as needed for its type.

“Feature recognition” may not be needed.

Story: Feature recognition of Philips Head screws
Assembly Features and Assembly Data Models

An assembly feature

Origin frame for a part

Typical CAD Model Based on World Coordinates

Validity of this model depends on shapes of parts being correct

Validity of this model does not depend on part shapes being correct

An Assembly Model Based on Connecting Assembly Features

Assembly model with variation based on error in feature location in part B
A Really Bad Example

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Source:
A Better Way, Based on Features and Frames
What The Different Models Do

- World coordinate model is good for drawing pictures of the nominal arrangement; can find interferences based on errors in the nominal but can’t help you find out why they happened.
- Chained model is good for capturing relational information and design intent, and can trace effects of variation from the nominal; won’t necessarily find interferences because it forces things to be “assembled”; can help you find out why things don’t fit.
Assembly Modeling in CAD

Parts can be defined with mating features on them. Features can be mated directly.
An assembly database builds up automatically.
Assembly knowledge can be accessed.
Information in Assembly Models

• What parts mate to what parts
• What features define the mates and where they are on the parts
• What interfaces must be controlled, plus a formal way of describing them
• Constraints and rule-checking
  – about assembly in the small
  – about assembly intent in terms of features
  – about assembly in the large, including alternate parts
• It is a completely abstract and general model based on connectivity
• Geometry is an attribute of the parts
Example Assembly Data Model

DECLARED ASSEMBLY FEATURE

ON PART_____ (text)

TYPE NAME____SPECIFIC NAME____

LOCATION ON PART _____ (4x4)
   LOCAL ESCAPE DIRECTION____
   (DEFAULT: Z AXIS)
   TOLERANCES

GEOMETRY____
   PARAMETERS________
   TOLERANCES____

OPTIONAL: FEATURES IT CAN MATE TO
   CONSTRAINTS

MATED TO FEATURE___ ON PART ___
   CASE... (other parts in other circumstances)
Seeker Head

Image removed for copyright reasons.
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Seeker Liaison Diagram

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Source:
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Feature-based Design Video

• Made at Draper in 1990
• Illustrates a bottom-up approach
• First demo of integrated design of assembly tools hooked to a CAD system
  – parts designed with mating features
  – parts joined by connecting the features
  – liaison diagram constructed automatically
  – assembly data model passed to CAE routines for assembly sequence, assembly system design, and economic analysis