Variation in Assemblies

• Goals of this class
  – Understand how to represent variation at the assembly level
  – Link models to models of variation at the part level
  – Link models to nominal chain of frames
  – Get an idea of statistical process control and statistical tolerancing vs worst case tolerancing
History of Tolerances in Assemblies

- Statistical process control and statistical tolerancing permit a bet on interchangeability
- Coordination makes a comeback as functional build
- Quality today is more than interchangeability
  - The product almost certainly will “work”
  - Real quality means
    - Durability
    - Reliability
    - Low noise, vibration, etc
  - All are associated with low running clearances, fine balance, etc., all requiring closer tolerances, robust design, etc.
Recent Gains in Productivity

• Wall Street Journal, Sept 28, 00, p 2:
  – “Many analysts may have overlooked one of the main drivers of the productivity gains of the last five years: machine tools… more than half the increase has occurred outside the computer, software, and telecommunications sectors...
  – Greater precision in machine tools has helped cut the energy use of air conditioners by 10% between 1990 and 1997 … using a new kind of compressor whose manufacture required … precision down to 10 microns”
  – One of the principal reasons for more reliable automobile transmissions ...
  – Open architecture, digital programming, ceramic tool bits are part of the picture
KC and Variation

- A KC is delivered when it achieves its desired value within some specified tolerance
- This puts the emphasis on behavior at the assembly level, not the part level
- To deliver the KC we must
  - Design the chain and the location strategy at each joint in the chain
  - Ensure that variation at the part level does not propagate to the assembly level and upset delivery of the KC
  - This means understanding variation in each of the joints in the chain
Logic Tree of Control of Variation

Main Decision

We can’t meet assembly KCs by controlling part variations

Functional Build

Selective Assembly

Belief in Coordination

Adjustment

We can meet assembly KCs by controlling part variations

Build to Print

Belief in “systems”

Statistical Process Control

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Two Kinds of Errors

- Change in process average (mean shift)
- Process variation around the mean (variance)
- These are different kinds of errors that require different approaches
  - The screwdriver workstation-RCC story
  - Whitehead’s reason for using kinematic design: to eliminate variation (“erratic errors”)
  - Taguchi’s systematic approach
    - Correct mean shift first, then reduce variation
    - The other sequence makes sense, too
Basic Definitions

Mean $\propto$

Standard Deviation $\sigma$

$\pm 3\sigma = 99.73\%$ of all events if Gaussian
Precision vs Accuracy

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Consistent but consistently wrong  Right on the average
Best Situation

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Source:
Models of How Errors Accumulate in Assemblies

- Worst case tolerancing essentially tracks the max error, treats it as mean shift, and ignores the variance
  - Assumes all errors are at their extremes at the same time
  - This is deterministic, not statistical
  - Errors accumulate linearly with the number of parts

- Statistical tolerancing (AKA root sum square) tracks the variance and assumes zero mean shift
  - Assumes errors are distributed randomly ± between limits
  - Errors accumulate with SQRT of number of parts but only if the average part error = the desired nominal dimension!
    (i.e., average part (& assy) error = 0.)
Stapler Variations

TOP VIEW
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FRONT VIEW
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Worst Case

Statistical
Logic Tree of Statistical and Worst-Case Tolerancing

Tolerance method

Worst case

Errors grow with $N$

Throws away parts that stat tol keeps

Statistical

Errors grow with $\sqrt{N}$ IF $\text{mean} = \text{nominal}$

Process capability measures adherence of process mean to nominal

Statistical Process control keeps process on its mean

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How Variations Accumulate

• Assumptions
  – All variations are described by the same distribution
  – All variations are independent of each other
  – The SSN jackpot in Milwaukee

• Then:
  – The mean or average of a sum of such variations is the sum of their individual means

\[ E \left( \sum_{i=1}^{N} x_i \right) = \sum_{i=1}^{N} E[x_i] = NE[x_i] \text{ if all } E[x_i] \text{ are the same} \]

• Actually this is true regardless of the above assumptions
  – The variance of the sum of such variations is the sum of their individual variances

\[ \sigma^2 \left( \sum_{i=1}^{N} x_i \right) = \sum_{i=1}^{N} \sigma^2[x_i] = N\sigma^2[x_i] \]
Total Error

Total Accumulated Error =
Accumulated Mean Errors
+ Accumulated Standard Deviation Errors

\[ = \sum_{i=1}^{N} \text{meanshift}_i + \sqrt{\sum_{i=1}^{N} (3\sigma_i)^2} \]

\[ = N * \text{meanshift} + \sqrt{N} * 3 * \text{stddev} \]

if all meanshifts and standard deviations are the same
What It Means

• Sums of zero-mean errors accumulate at the rate $\sqrt{N}$ because the + and - errors tend to cancel
• Sums of non-zero-mean errors accumulate at the rate $N$ because there are no cancellations
• Therefore, non-zero-mean errors accumulate much more rapidly than zero-mean errors
Root Sum Square (RSS) Method

Assume tolerance band $= \pm T$
Assume $T = 3\sigma$ of error distribution
Then accumulated error from $N$ of $T_i =$

$$\text{Accumulated error} = \sqrt{\sum_{i=1}^{N} T_i}$$

But this is really the entire accumulated error only if there is no meanshift. If there is meanshift then the RSS method will badly under-estimate the error.
Why the Mean Should Seek the Nominal

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How to survive Las Vegas

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Statistical and Worst Case Compared

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Statistical Process Control

• SPC aims to get a process in statistical control
  – The mean varies in a random way and stays within the mean control limits
  – The range varies in a random way and stays within the range control limits
• X-bar and R charts plot the mean and range of samples taken from production parts
• Deviations from the mean, consistent errors, and excess variation can be found quickly and eliminated before bad parts are made
• If a process is in control, then there is a chance that it can deliver parts without mean shift
Process Capability

• Every process has a mean and a “natural tolerance” range which should be tighter than the product needs.

• Process capability index $C_{pk}$ tracks whether the process range is within the $3\sigma$ tolerance band and the process mean is on the target nominal dimension.

\[
C_{pk} = \min \left[ \frac{USL - \bar{X}}{3\sigma}; \frac{LSL - \bar{X}}{3\sigma} \right]
\]

$USL$ = upper spec (tolerance) limit

$LSL$ = lower spec (tolerance) limit

$\bar{X}$ = mean of the process

$\sigma$ = variance of the process
Process Capability Cpk

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<table>
<thead>
<tr>
<th># of σ</th>
<th>% within ±# of σ</th>
<th>% outside σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.269%</td>
<td>31.731%</td>
</tr>
<tr>
<td>2</td>
<td>95.450%</td>
<td>4.550%</td>
</tr>
<tr>
<td>3</td>
<td>99.730%</td>
<td>0.270%</td>
</tr>
<tr>
<td>4</td>
<td>99.994%</td>
<td>0.006%</td>
</tr>
</tbody>
</table>
Process Control & Capability

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Effect of Mean Shift

Range | % Outside
---|---
± 1 sigma | 31.74%
± 2 sigma | 4.54%
± 2.5 sigma | 1.24%
± 3 sigma | 0.27%

0.00167 shift in each part

3σ = 0.01732

.005 total shift in 3 parts = 0.5σ

3σ = 0.03

2.5σ

1.24% will exceed 0.03

Cpk = 1.00  Cpk = .833
How Could Non-zero Mean Variations Occur?

- For material removal processes, operators stop as soon as the part enters the tolerance zone from the high side.
- For material-adding processes, operators stop as soon as the part enters the zone from the low side.

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Source:
Statistical vs Worst Case - Summary

• Statistical is cheaper because looser tolerances are allowed at each part or feature (cost rises as tolerances are tightened)

• Worst case guarantees interchangeable parts, even at batch size of one

• Statistical achieves interchangeability with some probability < 1, enhanced if there is a big bin of parts to choose from. The likelihood of the worst combination actually being picked is pretty low and is usually easy to recover from.
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Source:
Math Models of Variation

- There are tolerance standards for designing single parts (ANSI Y 14.5 xx)
- There is no standard for tolerancing assemblies
- People just extend the part standard or finesse
- The following slides show how to use the 4x4 matrix method for both nominal part-part location modeling and variation modeling
Nominal and Varied KC Relationships

Each relationship, internal or external, nominal or varied, is described by a 4x4 transform
Inside a Car Engine

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Source:
Chain of Frames for Engine

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Source:
Valve and Lifter Use Selective Assembly

Statistical Tolerancing cannot deliver this KC

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Image removed for copyright reasons.
Pick and measure a shaft.
If it is a bit big, pick a big bearing to get clearance right.
If it is a bit small, pick a small bearing.
For this to work over a long stretch, there must be about the same number of big shafts as big bearings, and the same for small ones.
Assembly Process Tolerances

NOMINAL

SHOWING TOLERANCES

T4
T3
T2
T1

T0

TA

T4
T3
T2
T1

T0

TA

T4
T3 + DT3 (PARALLEL)
T2 + DT2 (SIZE)

Not OK!

OK!

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Nominal Mating of Parts

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Varied Part Location Due to Tolerances

The varied location of Part B can be calculated from the nominal location of Part A. This process can be chained to Part C, etc., including errors on Part B. It uses the same math as the nominal model.
Equations for Connective Models

Nominal

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Source:

Varied

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Source:
Error Transform (Linearized)

\[ DT = \begin{bmatrix} 1 & -\delta_z & \delta_y & dx \\ \delta_z & 1 & -\delta_x & dy \\ -\delta_y & \delta_x & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ T' = T \times DT \]
Angle Errors Are Important

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Please see:
Kedrosky, Paul S. "In the Fray: What the Duffers Can Teach Tiger Woods."
Transform Order is Important

This is a frame 1 event

\[ T = DT_{11'}T_{12} \]

Tell us what everything looks like from frame 1’s pov

This is a frame 2 event

\[ T = T_{12}DT_{22} \]

Moves the focus to frame 2 and converts frame 2 events to frame 1 coords.
Peg Position Error

$$T_{AD} = \text{trans}(3,2,4) * \text{roty}(\text{dtr}(90))$$

% $\text{dtr}$ converts degrees to radians

$$\begin{bmatrix}
0 & 0 & 1 & 3 \\
0 & 1 & 0 & 2 \\
-1 & 0 & 0 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

% First method

$$DZ = \text{errt}$$

$$DT_{AD1} = \text{trans}(0,0,DZ)$$

$$T_{AD1} = DT_{AD1}T_{AD}$$

% Second method

$$DX = \text{errt}$$

$$DT_{AD2} = \text{trans}(-DX,0,0)$$

$$T_{AD2} = T_{AD}DT_{AD2}$$

$$T_{AD2} = T_{AD}T_{DD'}$$
Peg Length Error

\[ T_{AD} = \text{trans}(3,2,4) \ast \text{roty}(\text{dtr}(90)) \]

% \text{dtr} converts degrees to radians

\[
\begin{bmatrix}
0 & 0 & 1 & 3 \\
0 & 1 & 0 & 2 \\
1 & 0 & 0 & 4 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

» DTpp=trans(0,0,randn*.003)
Hole Angle Error

$T_{EF} = \text{trans}\ (6, 0, 1)$

$$\begin{bmatrix}
1 & 0 & 0 & 6 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

$DT_{EE'} = \text{rot}_y\ (d\text{tr}(erra))$

$T_{E'F} = DT_{E'E}T_{EF}$
Final Variation at Assembly Level

Where does this point go?

\[ T_{DE} = \text{rot}_z(dtr(180)) \]
\[ = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

\[ T_{AF} = T_{AD}T_{DE}T_{EF} \]
\[ = \begin{pmatrix}
0 & 0 & 1 & 4 \\
0 & -1 & 0 & 2 \\
1 & 0 & 0 & 10 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

\[ T_{AF}' = T_{AD'}T_{D'E'}T_{E'F} \]
\[ T_{DE} = T_{DE} \]
MATLAB for Example

»TAD=trans(3,2,4)*roty(dtr(90))

TAD =

0 0 1 3
0 1 0 2
-1 0 0 4
0 0 0 1

»TAF=TAD*TDE*TEF

TAF =

0 0 1 4
0 -1 0 2
1 0 0 -2
0 0 0 1

»TDE=rotz(dtr(180))

TDE =

1 0 0 0
0 0 0 1

»TEF=trans(6,0,1)

TEF =

-1 0 0 0
0 -1 0 0
0 0 1 0
1 0 0 6
0 1 0 0
0 0 1 1
0 0 0 1

for i=1:10000

DTad=trans(randn*.003,0,0);
DTdpp=trans(0,0,randn*.003);
DTee=roty(dtr((randn*.003)));
TAF=TAD*DTad*DTpp*TDE*DTee*TEF;
m(i)=TAF(1,4);
end

»hist(m,200)
Result for 10000 Samples

Nominal Value
Constructing Matrix Representations of GD&T

The Actual Surface Must Lie Inside the Zone

Rule #1: The surface must be correct at max material condition.

This, too, is allowed unless flatness is specified.
Position and Angle are Coupled by Rule #1

$f_{\text{opt}}$ is derived by simulation to give the best Gaussian fit to the actual diamond
Tolerance Zones as 4x4 Transforms

Two dimensional example:

\[ T_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & D \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{1,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ \delta_x = f_{opt} \times 2T_s / L_y \]
\[ \delta_y = f_{opt} \times 2T_s / L_x \]
\[ \delta_z = 0 \]

\[ f_{opt} = 0.95 \]

\[ T_{0,2} = T_{0,1} \times T_{1,2} \times DT_{1,2} \]
Table removed for copyright reasons.

Source:
Two Ways to Use This Model

• Direct Monte Carlo simulation (can use Matlab)
  – randomly select dX, dY, δ. x, etc (eliminate parts that violate Rule #1) (use f_{opt} = 1) Assume that T_s = 3\sigma
    \[
    dX = \text{randn} \times T_s / 3; \quad \delta. x = \text{randn} \times 2 \times T_s / (3 \times LY)
    \]
  – multiply out all the Transforms
  – repeat many times, collect histograms

• Plug into a closed form solution (use f_{opt} to approximate Rule #1)
  – requires assuming a Gaussian distribution for each of the errors
  – delivers ellipsoids of location and angle error at the end of the transform chain
Logic Tree of Control of Variation

Main Decision

We can’t meet assembly KCs by controlling part variations

We can meet assembly KCs by controlling part variations

Functional Build

Selective Assembly

Belief in Coordination

Adjustment

Build to Print

Belief in “systems”

Statistical Process Control

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Functional Build

• Accept the fact that the mean cannot be brought to the desired value:
  – Too much trouble, time, or cost
  – Too much variation in the mean
  – Some other mean may be shifted the other way

• Coordinate the parts involved

• Keep an eye on things and make adjustments

• This is often done on tools and dies if there is one source (one die) for each part involved

• Then the focus is to control variation around the mean

• The SPC metric is then \( C_P \)
  \[
  C_P = \left[ \frac{USL - LSL}{6\sigma} \right]
  \]
$C_p$ Ignores Mean Shift

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Source:
Logic Tree for Tolerancing

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Source: