2.882 System Design and Analysis

February 16
What we’ll do today

- Project discussion
- Information content, Robustness
Term Project Overview

• Key dates
  – Today: Project topic discussion, kick-off
    [ ~ 6 wks]
  – April 4: Interim progress report
    [ ~ 5 wks]
  – May 11: Project presentation
  – May 16: Written project report due

• Deliverables
  – Conceptual design solution
  – AD/Complexity analysis
  – Presentation, report
Project Examples from the previous year

- Engine project
- CEV architecture project
Information content

- Design range
- System range
- Probability of success
- (Allowable) Tolerance
Design Range

• **Examples of “range” in FR statements**
  - Maintain the speed of a vehicle at a $x$ mph +/- 5mph
  - Ensure no leakage under pressure up to 100 bar

• **Specification for FR**
• Acceptable range of values of a chosen FR metric; Goal-post
• Different from “tolerance”
• Different from “operating range”
• Target value (nominal), Upper bound, Lower bound
System Range

- Response/performance in FR domain, resulting from the chosen ‘design’
  - Here, ‘design’ includes both a chosen set of DPs and the way they deliver/affect FRs
- Due to various factors such as the input (DP) variation, internal/external noise, etc., FR takes a range of values, forming a range
Information content

\[ P(FR) = \int_{dr^l}^{dr^u} f(FR) dFR \]

\[ I = -\log_2 P = -\log_2 P(FR) = -\log_2 \int_{dr^l}^{dr^u} f(FR) dFR \]
Example

\[
\begin{pmatrix}
FR1 \\
FR2
\end{pmatrix}
= \begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}
\begin{pmatrix}
DP1 \\
DP2
\end{pmatrix}
\]

Design range
FR1: [-0.5, 0.5]
FR2: [-2.0, 2.0]

Taesik Lee © 2005
Detecting change in system range

“Monitoring marginal probability of each FR is not only inaccurate but potentially misleading”

Example

\[
\begin{align*}
\{ FR1 \} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} DP1 \end{bmatrix} \\
\{ FR2 \} &= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} DP2 \end{bmatrix}
\end{align*}
\]

Design range

FR1: [-0.5, 0.5]  
FR2: [-2, 2]

Design parameter variation

Initial  
DP1: U[-1,1]  
DP2: U[0,1.5]  

After change  
DP1: U[-1,1]  
DP2: U[-1,1.6]
Joint p.d.f. \((FR_1, FR_2)\)

Design range

(a) (b) (c)

DP1: \([-1, 1]\)
DP2: \([0, 1.5]\)

Before DP2 change

<table>
<thead>
<tr>
<th></th>
<th>(p_{FR_1})</th>
<th>(p_{FR_2})</th>
<th>(p_{FR_1} \times p_{FR_2})</th>
<th>(p_{FR_1,FR_2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>0.5</td>
<td>0.9583</td>
<td>0.4792</td>
<td>0.5</td>
</tr>
<tr>
<td>After</td>
<td>0.5</td>
<td>0.9654</td>
<td>0.4827</td>
<td>0.499</td>
</tr>
</tbody>
</table>

After DP2 change
Allowable tolerance

- Defined for DP
- Tolerances that DPs can take while FRs still remaining completely inside design ranges
- Unconditional tolerance
- Conservative tolerancing

\[ \Delta DP_1 = \frac{\Delta FR_1}{A_{11}} \]
\[ \Delta DP_2 = \frac{\Delta FR_2 - |A_{21} \cdot \Delta DP_1|}{A_{22}} \]
Linear tolerancing vs. Statistical tolerancing

\[
\begin{bmatrix}
FR1 \\
FR2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0.4 & 1
\end{bmatrix}
\begin{bmatrix}
DP1 \\
DP2
\end{bmatrix}
\]

Linear tolerancing

Design range
FR1: [-0.6, 0.6]
FR2: [-1.8, 1.8]

Statistical tolerancing

\[3\sigma_{\text{FR1}} = 0.6 \Rightarrow \sigma_{\text{FR1}} = 0.2 \]
Therefore, \( \sigma_{\text{DP1}} = 0.2 \)

\[\text{Var(FR2)} = 0.4^2 \text{Var(DP1)} + \text{Var(DP2)} \]
Thus, \( \sigma_{\text{DP2}} = 0.5946 \)

\[3 \sigma_{\text{DP1}} = 0.6 \]
\[3 \sigma_{\text{DP2}} = 1.784 \]
Robustness

• In axiomatic design, robust design is defined as a design that always satisfies the functional requirements,

\[ \Delta FR_i > \delta FR_i \]

even when there is a large random variation in the design parameter \( \delta DP_i \).

• Two different concepts in robustness
  – Insensitive to ‘noise’
    • Information Axiom
    • Traditional robust design
  – Adaptive to change
    • Independence Axiom
Example: Measuring the Height of a House with a Ladder

\[ H + \delta H = \sin \theta L + L \cos \theta \delta \theta \]
\[ \delta H = L \cos \theta \delta \theta \]

What if \( L \) also has uncertainty?
$$\overrightarrow{FR} - \overrightarrow{FR}^* = \left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{n}} \right)_{\overrightarrow{n}=0} \delta \overrightarrow{n} + \left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}} \right)_{\overrightarrow{DP} = \overrightarrow{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*) + \left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{C}} \right)_{\overrightarrow{C} = \overrightarrow{C}^*} (\overrightarrow{C} - \overrightarrow{C}^*)$$

0. Assign the largest possible tolerance
0. Eliminate the bias ( $E[FR] = FR^*$ )

1. Eliminate the variation: SPC, Poka-Yoke, etc.
2. De-sensitize: Taguchi robust design
3. Compensate

$$\left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{C}} \right)_{\overrightarrow{C} = \overrightarrow{C}^*} (\overrightarrow{C} - \overrightarrow{C}^*) = -\left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{n}} \right)_{\overrightarrow{n}=0} \delta \overrightarrow{n} + \left( \frac{\partial \overrightarrow{FR}}{\partial \overrightarrow{DP}} \right)_{\overrightarrow{DP} = \overrightarrow{DP}^*} (\overrightarrow{DP} - \overrightarrow{DP}^*)$$
Robustness built into a system by design

Example: Design of Low Friction Surface
- Dominant friction mechanism: Plowing by wear debris

- System range (particle size) moves out of the desired design range
  ⇒ Need to re-initialize

N. P. Suh and H.-C. Sin, Genesis of Friction, Wear, 1981
Design of Low Friction Surface

- Periodic undulation re-initializes the system range


Two figures (6-part diagram and pair of graphs) removed for copyright reasons.