2.882
Complexity

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Complexity in AD

• Complexity
  “Measure of uncertainty in achieving the desired functional requirements of a system”
  – Difficulty
  – Relativity
  – Information
  – Ignorance
Four types of complexity in AD

Complexity
Measure of uncertainty in achieving FR

Does uncertainty change with time?

Time-independent Complexity
Real complexity
Imaginary complexity

Time-dependent Complexity
Combinatorial complexity
Periodic complexity

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Complexity: A measure of *uncertainty* in achieving the desired set of FRs of a system

- **Time-independent real complexity**
  “Measure of uncertainty when the *probability of achieving the functional requirements is less than 1.0* (because the common range is not identical to the system range)”

- **Time-independent imaginary complexity**
  “Uncertainty that arises because of the *designer’s lack of knowledge and understanding* of a specific design itself”

- **Time-dependent combinatorial complexity**
  “Time-dependent combinatorial complexity arises because in many situations, *future events cannot be predicted a priori*. … This type of time-dependent complexity will be defined as time-dependent combinatorial complexity.”

- **Time-dependent periodic complexity**
  “Consider the problem of scheduling airline flights. … it is periodic and thus *uncertainties created during the prior period are irrelevant*. … This type of time-dependent complexity will be defined as time-dependent periodic complexity.”
Time-independent Real Complexity

- **Time-independent real complexity**
  - caused by system range’s being outside of the design range.
  - Real complexity ~ Information content
  - Take \( u_i \) as a random variable
    \[
    u_i = \begin{cases} 
    1 & \text{(success) with } P(FR_i = \text{success}) \\
    0 & \text{(failure) with } 1 - P(FR_i = \text{success}) 
    \end{cases}
    \]
  - Information content:
    \[
    I(u_i = 1) = - \log_2 P(FR_i = \text{success})
    \]
Time-independent Imaginary complexity

- Imaginary complexity ~ Ignorance
- Ignorance causes complexity.
- Types of ignorance
  - Functional requirement
  - Knowledge required to synthesize(or identify) design parameters
  - Ignorance about the interactions between FRs and DPs
- $p$ (probability of selecting a right sequence)
  - For uncoupled design, $p = 1$
  - For decoupled design, $p = \frac{z}{n!}$
  - For coupled design, $p = 0$
Time-dependent complexity

• Time-dependency
  – Complexity \equiv \text{Uncertainty in achieving a set of FR}
  – Complexity is time-dependent if
    1) uncertainty (probabilistic) is time-dependent
       Time-varying system range
    2) behavior of FR is time-dependent
       \[ FR = FR(t) \]

• Combinatorial / Periodic complexity
  – Uncertainty increases indefinitely: combinatorial complexity
  – Uncertainty in one period is irrelevant to the next period: periodic complexity
Origins of complexity and reduction
Time-independent

• Minimize Real complexity by
  – Eliminating source of variation
  – Desensitizing w.r.t. variation
  – Compensating error

• Eliminate Imaginary complexity by
  – Achieving uncoupled design
  – Identifying design matrix
Time-dependent complexity

Combinatorial complexity $\Rightarrow$ Periodic complexity
Time-varying system range

- Detect changes in system range
- Prevent system range deterioration by design
- Bring the system range back into design range by re-initialization

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Prevent system range deterioration by design

By eliminating coupling between ‘turn’ and ‘grasp’, one can effectively delay system range deterioration.

Bring the system range back into design range: Re-initialization

Example: Design of Low Friction Surface
• Dominant friction mechanism: Plowing by wear debris

• System range (particle size) moves out of the desired design range
  ⇒ Need to re-initialize

Figure removed for copyright reasons.

N. P. Suh and H.-C. Sin, Genesis of Friction, Wear, 1981


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Design of Low Friction Surface

- Periodic undulation re-initializes the system range

Figures removed for copyright reasons.

Example: Scheduling of mfg system

Periodicity should be introduced & maintained to prevent the system from developing chaotic behavior
Problem description

- Fastest speed*
  70 seconds when X determines the system speed
  65-75 seconds when Y determines the system speed

- Objective
  Maximum utilization rate for the machine Y

- Constraint
  Transport from C to D must be immediate

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<table>
<thead>
<tr>
<th>Station</th>
<th>$PT_i$ or $CT_i$ (sec)</th>
<th>Number of machines</th>
<th>$M_vP_k$ (sec)</th>
<th>$M_vP_l$ (sec)</th>
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<tr>
<td>IN</td>
<td>-</td>
<td>1</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
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<td>a 30</td>
<td>1</td>
<td>5</td>
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<tr>
<td></td>
<td>b 40</td>
<td>1</td>
<td>5</td>
<td>5</td>
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<tr>
<td></td>
<td>c 50</td>
<td>1</td>
<td>5</td>
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<tr>
<td></td>
<td>d 80</td>
<td>2</td>
<td>5</td>
<td>5</td>
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<tr>
<td>Y</td>
<td>60±5</td>
<td>1</td>
<td>-</td>
<td>5</td>
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</tbody>
</table>

* Speed is measured by throughput time: shorter time means faster speed

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1st demand from Y
Y process strats
1st part ready at X
2nd part ready at X
3rd part ready at X
4th part ready at X
5th part ready at X
6th part ready at X
Time, t

1st demand from Y
Y process strats
1st part ready at X
2nd part ready at X
3rd part ready at X
4th part ready at X
5th part ready at X
6th part ready at X
Time, t

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$CT_Y : ... \text{60sec - 60sec - 60sec - 60sec - 55sec - ...}$

Figure 10. Steady state operation with 70 seconds sending period

CT\textsubscript{Y} : … 60sec - 60sec - 55sec - 65sec - 65sec …

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Average throughput time = \( \frac{(75+75+75+100)}{4} = 81.25 \)
• Single perturbation from subsystem Y causes incomplete period in downstream
• The system regains periodicity after the perturbation is removed but with undesirable performance
• Throughput time is 81.25 seconds in average
  – Slower than the system capability
Re-initialization scheme in scheduling

- Define a “renewal” event that imposes period

\[ u(0) = \{ u_0(t), u_1(t), \ldots, u_{k-1}(t), u_k(t), u_{k+1}(t), \ldots, u_N(t) \} \]
\[ = \{ 0, 0, \ldots, 0, 0, 0, \ldots, 0 \} \]
\[ \vdots \]
\[ u(T-\Delta) = \{ 1, 1, \ldots, 1, 0, 1, \ldots, 1 \} \]
\[ u(T) = \{ 1, 1, \ldots, 1, 1, 1, \ldots, 1 \} \]
\[ u(T+\epsilon) = \{ 0, 0, \ldots, 0, 0, 0, \ldots, 0 \} = u(0) \]

- Scheduling activity is confined within such a period with a goal of maintaining “periodicity”
  - Conditional renewal event

\[ t_{ini} = t_{request} \quad \text{if } t_{request} \geq 70 \text{ sec (FP}_X) \]
\[ t_{ini} = 70\text{sec} \quad \text{if } t_{request} < 70 \text{ sec} \]
• Each period is independent (memoryless)
Conclusion

- Breakdown of functional periodicity results in sub-optimal throughput rate
- Periodicity should be introduced & maintained to prevent the system from developing chaotic behavior
Example: Cell division

- A cell has a mechanism to coordinate cycles of two subsystems such that the overall periodicity is maintained
- Break-down of functional periodicity leads to anomaly of cell division and further chaotic behavior of the system
- Maintaining functional periodicity in the cell cycle is an important functional requirement for cell division
Overview of the Cell Cycle

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* Figure taken from Molecular Biology of the Cell, Alberts, Garland Science
Chromosome cycle & Centrosome cycle

Figure removed for copyright reasons.

* Figure taken from Molecular Biology of the Cell, Alberts, Garland Science
Importance of the correct number of chromosomes and centrosomes

• Centrosomal abnormalities
  – Chromosome missegregation
  – Aneuploidy

Figure removed for copyright reasons.

Figure 2 in Nigg, E. A. "Centrosome abberation: cause or consequence of cancer progression?" *Nature Reviews Cancer* 2 (2002): 815-825.

* Figure taken from http://www.sivf.com.au/chromosomes.htm

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Functional periodicity

Figure removed for copyright reasons.
See Figure 1 in Nigg, E. A. "Centrosome aberration: cause or consequence of cancer progression?" Nature Reviews Cancer 2 (2002): 815-825.
Mechanism

Cdk2 initiates both cycle ensuring one level of synchronization

Cdk2 (S-Cdk)

Mitogen-dependent mechanism (Extracellular signal)

G1/S-

Cdk

S-Cdk

Hct1–APC

inactivate

mutually inhibit

mutually inhibit

E2F

S-cyclin

Rb

mutually inhibit

G1/S-cyclin

inactivate

G1-Cdk

inactivate

promote accumulation

DNA replication checkpoint ensures completion of chromosome duplication

Chromosome duplication

Replicating DNA

Firing origin of replication

Check for completion of DNA replication

Centrosome cycle lacks a mechanism to check its completion

Centrosome duplication

Centriole split

Duplicate centrosome

Check for completion

DNA replication

Check for completion of DNA replication

Cytokinesis

M-Cdk activity

Cdc20-APC activity

Hct1-APC activity

M phase

G1 phase

Cytokinesis

Anaphase

Telophase

Metaphase

Prometaphase

Prophase

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Conclusion

- A cell has a mechanism to coordinate cycles of two subsystems such that the overall periodicity is maintained

- Maintaining functional periodicity in the cell cycle is an important functional requirement for cell division
  - Can pose questions with new perspective
Time-dependent FR

- Functional periodicity
- \( u(t) = \{ u_1(t), u_2(t), \ldots, u_N(t) \} \)

  - **Periodic** – There exist \( T_i \) s.t. \( u(T_i) = u(T_j) \) with regular transition pattern
  - **Semi-periodic** – There exist \( T_i \) s.t. \( u(T_i) = u(T_j) \) without regular transition pattern
  - **Aperiodic** – None of the above

![Diagram](image_url)

(a) Periodic  
(b) Semi-periodic  
(c) Aperiodic

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Uncertainty and functional periodicity

• \( P_s(t) = P(u(t) = u^*(t)) \)
  – For periodic & semi-periodic \( FR(t) \), \( P_s \) returns to one at the beginning of a new period

• Predictability of \( FR \)
  – (Periodicity) \( \rightarrow \) (Predictability)
  – (Unpredictability) \( \rightarrow \) (Aperiodicity) \( \iff \)
    \( \sim \) (Aperiodicity) \( \rightarrow \) \( \sim \) (Unpredictability)

• Uncertainty in current period is independent of a prior period only if the initial state is properly established