1. Let $a$ be a positive real number, and let the sequence of real numbers $x_i$ be given by

\[ x_0 = 1, \quad x_{i+1} = \frac{1}{2} \left( x_i + \frac{a}{x_i} \right), \]

for $i = 0, 1, 2, 3, \ldots$

The value $x_i$ will converge to $\sqrt{a}$ as $i \to \infty$ Write a program that reads in the value of $a$ interactively and uses this algorithm to compute the square root of $a$.

Test your program as you vary the maximum number of iterations of the algorithm is increased from 1, 2, 3, … and determine how many significant digits of precision that you obtain for each. How many iterations are necessary to reach the machine precision of matlab?

2. Write a program to evaluate $e$ by the series:

\[ e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \ldots \]

Test your program as you increase the number of terms in the series. Determine how many significant digits of precision that you obtain in your answer as a function of the number of terms in the series. How many terms are necessary to reach machine precision?

3. Consider the function $x \sin(x) - 1$.

(a.) How many roots does this function have in the interval $[0, \pi]$?

(b.) Write a matlab program to find the root(s) using Newton-Raphson iteration with appropriate starting values.

(c.) Make a graph of relative error vs. iteration step for all roots.

(d.) How many iterations are needed to reach an error of less than $10^{-8}$?