Lecture 6 Thermionic Engines

• Review
• Richardson formula
• Thermionic engines
• Schottky barrier and diode
• pn junction and diode
• discussion
Review: Fermi-Dirac Distribution

- Average number of electrons in the state

\[ f = \sum_{n=0,1} n A e^{-(E-\mu)/(k_B T)} = \frac{1}{\exp\left(\frac{E-\mu}{k_B T}\right) + 1} \]

At T=0K, \( \mu \) is called Fermi level, \( E_f \)

\[ f=1 \text{ for } E<\mu \]
\[ f=0 \text{ for } E>\mu \]
Review: Electron Density

\[ E - E_c = \frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m} \]

\[ N = 2 \sum_{N_x/2}^{N_x/2} \sum_{N_y/2}^{N_y/2} \sum_{N_z/2}^{N_z/2} f(E, T) = \frac{2V}{8\pi^3} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \exp \left[ -\frac{E - \mu}{k_B T} \right] \]

\[ n = \int_{E_c}^\infty D(E) f(E, \mu, T) dE \]

\[ n = 2 \left( \frac{2\pi m^* \kappa_B T}{\hbar^2} \right)^{3/2} \exp \left( -\frac{E_c - \mu}{k_B T} \right) = N_c \exp \left( -\frac{E_c - \mu}{k_B T} \right) \]
Different Solids

- Metal
- Insulator
- n-type Semiconductor
- p-type Semiconductor

Energy vs. $k/(\pi/a)$

Fermi Level

Gap $E_g$

Donor Energy Level

Acceptor Energy Level

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Work Function and Affinity

Electron

Vacuum

Work Function $W$

Fermi Level $E_f$

Metal Electrodes

Current

Light

Affinity

$E_c$

$E_f$

Bandgap $E_g$

$E_v$
Electrons Just Outside the Metal

\[ E - \mu - W = \frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m} \]

Electron particle Flux Leaving Surface

\[ J_p = \sum_{k_x > 0} \sum_{k_y = -\pi/a}^{\pi/a} \sum_{k_z = -\pi/a}^{\pi/a} v_x f \]

\[ = \frac{2V}{8\pi^3} \int_0^{\pi/a} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \frac{\hbar k_x}{m} f \]
Electron Flux Out of Surface

\[
J_p = \frac{2}{8\pi^3} \int_0^{\pi/a} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \frac{\hbar k_x}{m} \frac{1}{m} \exp\left(-\frac{E - \mu}{k_B T}\right) + 1
\]

\[
\approx \frac{2}{8\pi^3} \int_0^{\pi/a} \int_{-\pi/a}^{\pi/a} \int_{-\pi/a}^{\pi/a} dk_x dk_y dk_z \frac{\hbar k_x}{m} \exp\left(-\left[\frac{\hbar^2 (k_x^2 + k_y^2 + k_z^2)}{2m}\right]\frac{1}{k_B T}\right)
\]

\[
= \frac{m}{2\pi^2 \hbar^3 (k_B T)^2} \exp\left(-\frac{W}{k_B T}\right)
\]
Electrical Current Density

\[ J_e = \frac{em}{2\pi^2\hbar^3} (k_B T)^2 \exp\left( -\frac{W}{k_B T} \right) = AT^2 \exp\left( -\frac{W}{k_B T} \right) \]

Richardson Formula or Richardson-Dushman Equation
For Thermionic Emission

\[ A = \frac{emk_B^2}{2\pi^2\hbar^3} = 120 \frac{A}{cm^2 K^2} \]
Richardson Constant

\[ J_e = (1-r)AT^2 \exp\left( -\frac{\phi}{k_B T} \right) \]
More General

O.W. Richardson
1928 Nobel Prize
Contact Potential

\[ W_S = \chi + q\varphi_n \]

\[ E_0 \]
\[ WM \]
\[ E_F \]

\[ E_0 \]
\[ WS \]
\[ \chi \]
\[ E_F \]

\[ q\varphi_n \]

\[ E_0 \]
\[ \chi \]
\[ E_F \]

\[ q\phi_{bi} \]

\[ E_0 \]
\[ WM \]
\[ E_F \]

\[ q\varphi_{Bn} \]

\[ E_v \]

a) metal and semiconductor far apart

\[ \varphi \neq \varphi' \]

\[ -e(\varphi - \varphi') = W - W' = -eV_c \]

Courtesy of Jesús del Alamo. Used with permission.
At equilibrium, \( J = J_c - J_a = 0 \)

\[
J_c = AT^2 \exp\left(-\frac{W_c + eV_c}{k_B T}\right)
\]

\[
J_a = AT^2 \exp\left(-\frac{W_a}{k_B T}\right)
\]

\( -eV_c = (W_a - W_c) \)
Thermionic Generator

\[ J_c = A T_c^2 \exp \left( - \frac{W_c + eV_c + eV_o}{k_B T_c} \right) \]

\[ J_a = A T_a^2 \exp \left( - \frac{W_a}{k_B T_a} \right) \]

At equilibrium, \( J = J_c - J_a = 0 \)

\(-eV_o = \mu_a - \mu_c\)
Thermionic Generator

\[ AT_c^2 \exp\left(-\frac{W_c + eV_c + eV_o}{k_BT_c}\right) - AT_a^2 \exp\left(-\frac{W_a}{k_BT_a}\right) = 0 \]

\[ 2 \ln\left(\frac{T_c}{T_a}\right) = \frac{W_c + eV_c + eV_o}{k_BT_c} - \frac{W_a}{k_BT_a} \]

Open Circuit Voltage:

\[ V_o = \frac{W_a}{e} \left(\frac{T_c}{T_a} - 1\right) + 2 \frac{k_BT_c}{e} \ln\left(\frac{T_c}{T_a}\right) \]
Under Operation

\[ J = AT_c^2 \exp\left(-\frac{W_c + eV_c + eV}{k_BT_c}\right) - AT_a^2 \exp\left(-\frac{W_a}{k_BT_a}\right) \]

\[ = AT_c^2 \exp\left(-\frac{W_a + eV}{k_BT_c}\right) - AT_a^2 \exp\left(-\frac{W_a}{k_BT_a}\right) \]

Power Output

\[ P = JV \]
Heat Transfer: Electron Heat Flux

Kinetic Energy:

\[ J_{k,c} = \frac{2}{8\pi^3} \int_0^{\pi/a} \int_{-\pi/a}^{\pi/a} \int dk_x dk_y dk_z \left( \frac{\hbar^2 k_x^2}{2m} \right) \frac{1}{\exp \left( \frac{E - \mu}{k_B T} \right) + 1} \]

\[ = \frac{2k_B T_c}{e} J_c \]

Total Heat from Cathode to Anode:

\[ J_{q,c} = \left( \frac{W_a}{e} - V + \frac{2k_B T_c}{e} \right) J_c \]

Total Heat from Cathode to Anode:

\[ J_{q,a} = \left( \frac{W_a}{e} + \frac{2k_B T_a}{e} \right) J_a \]
Net Heat Input

\[ Q_h = J_{q,c} - J_{q,a} + Q_{\text{rad}} + Q_{\text{cond}} \]

- **Q\text{rad}** --- Radiation heat transfer between cathode and anode
- **Q\text{cond}** --- Conduction heat loss through leads, and gas in between
\[\eta = \frac{Power}{Q_h}\]

When radiation is neglected

\[eV_a = W_a\]

S.W. Angrist,
Direct Energy Conversion,
Allyn and Bacon, Boston, 1965

Figure by MIT OpenCourseWare.
Experimental Demonstration


Image removed due to copyright restrictions. Please see Fig. 3 in Hatsopoulos, George N., and Joseph Kaye. "Measured Thermal Efficiencies of a Diode Configuration of a Thermo Electron Engine." *Journal of Applied Physics* 29 (1958): 1124-1125.
Real Device Issues

- Large work function, high temperature
- Cesium to reduce space charge, but reliability problem
- Vacuum operation or plasma operation (filled with gas)
Recent Trends

- Negative electron affinity materials
- Small gap devices
- Solid-state thermionics

Smith et al., Diamond and related materials, 15, 2082 (2006)

Schottky Barrier

- Work Function $W$
- Fermi Level $E_f$
- Bandgap $E_g$
- Vacuum

Metal

N-type Semiconductor

Schottky Barrier

Interface

Metal

Semiconductor

Vacuum

$E_c$

$E_v$

$E_f$

$\Delta$

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For 2.5 Solar/Energy Conversion

Electrical/Energy Conversion
P-type Schottky Barrier

Metal

Semicon

http://w3.ualg.pt/~pjotr/Images/Schottky.png

Courtesy of Peter Stalllinga. Used with permission.
Schottky Diode

Richardson Formula

\[ J = A T^2 \exp \left( -\frac{\Delta}{k_B T} \right) \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right] \]
pn Junction

Electron Energy Increasingly Negative

Build-In Potential

V_{bi}

p-Type

n-TYPE

Space Charge Region

Hole Energy Increasingly Positive

n-Type

p-Type

Space Charge Region
pn Junction Basics

- For nondegenerate semiconductor

\[
\begin{align*}
n &= N_c \exp\left(-\frac{E_c - \mu}{k_B T}\right) \\
p &= N_v \exp\left(-\frac{\mu - E_v}{k_B T}\right) \\
pn &= N_c N_v \exp\left(-\frac{E_g}{k_B T}\right) = n_i^2
\end{align*}
\]
pn Junction Basics: Built-in Potential

Electron Energy Increasingly Negative

Build-In Potential

\[ V_{bi} \]

p-Type

n-TYPE

Space Charge Region

Hole Energy Increasingly Positive

\[ eV_{bi} = E_{c,p} - E_{c,n} = E_g - k_B T \ln \left( \frac{N_v N_c}{N_A N_D} \right) = k_B T \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

\[ N_D = N_c \exp \left( - \frac{E_{c,n} - \mu}{k_B T} \right) \]

\[ E_{c,n} - \mu = k_B T \ln \left( \frac{N_c}{N_D} \right) \]

\[ E_{c,p} - \mu = E_g - k_B T \ln \left( \frac{N_v}{N_A} \right) \]

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pn Junction Basics: Space Charge Region

Electron Energy Increasingly Negative

Build-In Potential

$V_{bi}$

p-Type

Hole Energy Increasingly Positive

Space Charge Region

One Side Only

$$w = \sqrt{\frac{2\varepsilon_s}{e} \left( \frac{N_A + N_D}{N_A N_D} \right)} V_{bi}$$

Debye Length

$$w = \sqrt{\frac{2\varepsilon_s}{e N_B}} V_{bi}$$

Debye Length

$$w = \sqrt{\frac{2\varepsilon_s}{e^2 N_B}} k_B T$$
pn Junction I-V

\[ J = J_s \left( e^{\frac{eV}{k_BT}} - 1 \right) \]

Saturation Current

\[ J_s = eN_c N_v \left( \frac{1}{N_A} \sqrt{\frac{a_h}{\tau_h}} + \frac{1}{N_D} \sqrt{\frac{a_e}{\tau_e}} \right) \exp \left( -\frac{E_G}{\kappa_B T} \right) \]

- \( a_h \) --- hole diffusivity (m\(^2\)/s)
- \( a_e \) --- electron diffusivity
- \( \tau_h \) --- hole recombination time
- \( \tau_e \) --- electron recombination time
Compare Schottky diode and pn diode

\[ J = J_s \left( e^{\frac{eV}{k_B T}} - 1 \right) \]

\[ J_s = A T^2 \exp \left( -\frac{\Delta}{k_B T} \right) \]

\[ J_s = e N_c N_v \left( \frac{1}{N_A} \sqrt{\frac{a_h}{\tau_h}} + \frac{1}{N_D} \sqrt{\frac{a_e}{\tau_e}} \right) \exp \left( -\frac{E_G}{k_B T} \right) \]
Current and Energy Distribution

- Current and Energy Distribution
- hole diffusion
- electron diffusion
- n-type
- p-type
- Energy Source Distribution
- Positive
- Negative
2.997 Direct Solar/Thermal to Electrical Energy Conversion Technologies
Fall 2009

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