Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

1  Smoke Detector Operation

In these problems, consider the decay of $^{241}$Am, the isotope used in ionization chamber-type smoke detectors.

1. Write the two possible types of decay reactions for $^{241}$Am, and state which decay processes (and competing processes) may be possible for each general type of reaction. You don’t have to address every single energy level, there are dozens! Just group them into categories.

2. Now consider only the three most likely alpha decay energies of $^{241}$Am.

   (a) Draw a complete energy level diagram showing alpha decay to these three energy levels, and any possible, successive decays to the ground state.

   (b) For each of the initial alpha particle energies, separately sketch a hypothetical photon (gamma plus x-ray) spectrum that you would expect to observe. You may want to use the NIST X-Ray Transition Energy Database to help generate your answer.

3. It is clear that $^{241}$Am produces a few types of radiation at many different energies. Do you expect the alpha particles, the gamma rays, or the x-rays to be responsible for producing the largest number of ions in a fixed space (like in a smoke detector), and why?

2  Medical Isotope Physics

In these problems, consider the decay of $^{99}$Mo, a crucial medical isotope widely used in imaging and diagnosis procedures.

1. Calculate the Q-value for the decay of $^{99}$Mo using the binding energies of the initial and final nuclei, and any other information that you need.

2. You may have noticed that $^{99}$Mo is an unstable isotope. Which nuclear reactions could create $^{99}$Mo? Write the nuclear reactions for these processes, and calculate their Q-values to justify your answer.

3. For the most common $^{99}$Mo production method which does not arise from spontaneous radioactive decay, answer the following questions:

   (a) Assuming the incoming particle was at room temperature to begin with (you should find its kinetic energy), what are the possible recoil kinetic energies of the $^{99}$Mo produced?

   (b) How will the recoil energy and the outgoing radiation energy change if the incoming radiation had a kinetic energy of 2 MeV?
3 Q-Equation Derivation

1. Consider the kinematic system in Figure 8.1 (p. 142). In the laboratory coordinate system (LCS), show the origin of equation 8.4 (the Q-equation) by conserving energy, x-momentum, and y-momentum of all particles involved in the collision.

2. How does the Q-equation change when considering particles with relativistic speeds? Write the new, complete Q-equation in this case.

4 Allowable Nuclear Reactions

For these problems, determine whether the following reactions would be allowed, and answer the additional questions.

1. Which of the following decay methods are energetically allowable from the ground state of $^{216}\text{At}$? Back up your reasoning with an energetic argument.
   
   (a) Alpha decay
   (b) Beta decay
   (c) Positron decay
   (d) Electron capture
   (e) Isomeric transition
   (f) Spontaneous fission
      
      i. Can you find an instance where this particular one is energetically allowable?
      
      ii. Why do you think it's never observed?

2. For the reactions which are allowed, write the full nuclear reaction in each case, and draw a graph of the energy spectrum you would expect to see from each released form of radiation, including secondary ejections of particles or photons.

3. For the reactions which are not allowed, under which conditions could they be allowable? In other words, how would you insert energy into the system to make them allowed, and how much?