Quiz Instructions: Answers can be given symbolically or graphically, no calculation is necessary. No calculators, devices, or anything else allowed, except one double-sided, 8.5 x 11 inch sheet of paper. Define any intermediate variables which you need to complete the problems. Generous partial credit will be given for showing correct methodology, even if the solution is not given.

1 (48 points) Short Answers, 6 points each

Each of these problems can be solved with one sentence, one equation, and/or one graph/picture.

1.1 Which types of isotopes can decay by both positron ($\beta^+$) and beta ($\beta^-$) decay? Be specific about their nuclear structure.

1.2 Write a necessary and sufficient inequality that describes when a nuclear reaction is energetically allowed to occur.

1.3 What is the least likely emitted anti-neutrino energy during beta decay?

1.4 Which nuclear decay type(s) could directly or indirectly cause Auger electron emission to occur, and why?

1.5 Why can't we round atomic masses when performing energetics calculations? Give an example where a rounding error would be severe.

1.6 What is the kinetic energy of a nucleus following alpha decay?

1.7 Why must uncertainties be added in quadrature, not as a simple sum? Write the formula for the combined uncertainty of two experiments together, each with their own uncertainty.

1.8 Suppose you measure $10^5$ gamma ray counts from a $^{40}\text{K}$ source in 100,000 seconds, using a 0.01% efficient detector. What is the activity of your $^{40}\text{K}$ source in Becquerels (Bq), and what is the uncertainty of the activity? Ignore background counts. Use the following diagram to help you:
2 (26 points) Breeding Reactor Fuel

Reminder: Answers can be given symbolically or graphically, no calculation is necessary.

Reactors can breed their own fuel, by converting $^{238}\text{U}$, a fertile element, to $^{239}\text{Pu}$, a fissile element like $^{235}\text{U}$. $^{238}\text{U}$ can capture a neutron, which then decays via beta decay to $^{239}\text{Np}$, which decays via beta decay to $^{239}\text{Pu}$. Key data for this problem include:

\[
\begin{align*}
\sigma_{f,U-235} &= 500 \text{b} \\
\sigma_{f,Pu-239} &= 500 \text{b} \\
\sigma_{c,U-238} &= 5 \text{b} \\
t_{1/2,U-239} &= 1.5 \times 10^3 \text{sec} \\
t_{1/2,U-238} &= 2 \times 10^5 \text{sec} \\
\Phi &= 10^{14} \frac{n}{\text{cm}^2\text{s}}
\end{align*}
\]

2.1 (5 points) Write the complete nuclear reactions or radioactive decay reactions for each of the following isotopes: $^{235}\text{U}$, $^{238}\text{U}$, $^{239}\text{U}$, $^{239}\text{Np}$, $^{239}\text{Pu}$.

2.2 (10 points) Draw a graph of the total amount of fissile material in the reactor versus time. You may want to start by writing differential equations describing the production (by neutron capture) and destruction (radioactive decay and/or fission) of each isotope.

2.3 (11 points) Write an expression for the breeding ratio, or the rate of fissile material creation vs. fissile material burning.

3 (26 points) Decay Chain Diagrams

For these problems, consider the decay of $^{50}\text{V}$, which decays by multiple paths as shown below:

3.1 (8 points) Write the complete nuclear reactions for all possible decays shown, assuming that $^{50}\text{Ti}$ and $^{50}\text{Cr}$ are stable isotopes.

3.2 (10 points) Draw complete photon and electron spectra which would be observed from the decay of $^{50}\text{V}$.

3.3 (6 points) Why do nuclei like $^{50}\text{V}$ decay by either mechanism shown here, while most isotopes only have one mode of decay?