MHD

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What is MHD

• MHD stands for magnetohydrodynamics

• MHD is a simple, self-consistent fluid description of a fusion plasma

• Its main application involves the macroscopic equilibrium and stability of a plasma
Equilibrium and Stability

• Why separate the macroscopic behavior into two pieces?
• Even though MHD is simple, it is still involves nonlinear 3-D + time equations
• This is tough to solve
• Separation simplifies the problem
• Equilibrium requires 2-D non-linear time independent
• Stability requires 3-D+ time, but is linear
• This enormously simplifies the analysis
What is Equilibrium

• We must design a magnet system such that the plasma is in steady state force balance
• So far tokamaks are the best design
• The spherical torus is another option
• The stellarator is yet another option
• Each can provide force balance for a reasonably high plasma pressure
What is Stability?

• In general a plasma equilibrium may be stable or unstable

• Stability is good!

• Instability is bad!
Examples of Stability

Figure by MIT OCW.
Effects of an MHD Instability

- Usually disastrous
- Plasma moves and crashes into the wall
- No more fusion
- No more wall (in a reactor)
- This is known as a major disruption
The Job of MHD

• Find magnetic geometries that stably confine high pressure plasmas

• Large amount of theoretical and computational work has been done

• Well tested in experiments
Current status of fusion MHD

• Some say there is nothing left to do in fusion MHD
  a. The theory is essentially complete
  b. Computational tools are readily available
  c. Used routinely in experiments

• There is some truth in this view

• But not really – there are major unsolved MHD problems
What do we know?

- MHD equilibrium in 2-D and 3-D
- MHD stability pressure limits ($\beta$)
- MHD stability current limits ($q_*$)
- MHD stability shaping limits ($\kappa$)
- Plasma engineering coil design
What don’t we know?

- Resistive wall mode
- Plasma rotation
- High bootstrap current
- Edge localized modes
- Neoclassical tearing modes
Goals of the Talk

• Tell the story of fusion

• Show how the unsolved MHD problems fit into the story

• Discuss the paths to a perhaps happy ending
Outline of the Talk

• Design a tokamak fusion reactor
• Describe the current status of the tokamak
• Describe one crucial unsolved problem
• Show how the unsolved MHD problems enter the picture
• Show how we might proceed into the future
A Tokamak Fusion Reactor

• Based on the $D-T$ reaction

\[ D + T \rightarrow n + \alpha + 17.6 \text{ MeV} \]

• *neutrons* escape and produce heat and electricity

• *alphas* stay confined and balance $\chi$ heat loss
A Tokamak Fusion Reactor

- TF coils produce stabilizing toroidal magnetic field
- PF coils produce equilibrium poloidal magnetic field
- OH transformer induces toroidal plasma current
Design Strategy

- Minimize the Cost/Watt subject to 
- Nuclear physics constraints: $\lambda_{\text{mfp}} \approx \text{few cm}$ 
- Magnet Constraints: $B_{\text{max}} = 13 \ T$, $\sigma_{\text{max}} = 300 \ MPa$ 
- Wall loading constraint: $P_w = P_{\text{neutron}}/A = 4 \ MW/m^2$ 
- Output power constraint: $P_E = \eta P_{\text{fusion}} = 1000 \ MW$ 
- Self sustaining constraint: $P_\alpha = P_{\text{loss}}$ 
- Determine: $a, R_0, T, \rho, n, \tau_E$ 
- Design is almost independent of plasma physics and MHD
Minimum Cost/Watt

• Minimum cost/watt is proportional to the volume of reactor material per watt of electricity

• Volume dominated by the blanket/shield and TF coils

• Minimize \( \frac{V}{P_E} \)

\[
\frac{V}{P_E} = \frac{2\pi^2 R_0 \left[ (a + b + c)^2 - a^2 \right]}{P_E}
\]
Design

- Nuclear physics constraints: \( b = 1.2 \ m \)
- Magnet Constraints: \( c = 0.25(a + b) \)
- Optimize \( V/P_E(a) \): \( a = 2 \ m, \ c = 0.8 \ m \)
- Wall loading constraint: \( R_0 = 0.04P_E / aP_W = 5 \ m \)
- Output power constraint: \( T = 15 \ keV, \ p = 7 \ atm \)
- Self sustaining constraint: \( \tau_E = 1.2 \ sec \)
## Comparisons of Parameters

<table>
<thead>
<tr>
<th>Reactor</th>
<th>Steady State ITER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 2 \text{ m}$</td>
<td>$a = 2.3 \text{ m}$</td>
</tr>
<tr>
<td>$R_0 = 5 \text{ m}$</td>
<td>$R_0 = 8.7 \text{ m}$</td>
</tr>
<tr>
<td>$T = 15 \text{ keV}$</td>
<td>$T = 13 \text{ keV}$</td>
</tr>
<tr>
<td>$p = 7 \text{ atm}$</td>
<td>$p = 5 \text{ atm}$</td>
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<tr>
<td>$n = 1.2 \times 10^{20} \text{ m}^{-3}$</td>
<td>$n = 1.0 \times 10^{20} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>$\tau_E = 1.2 \text{ sec}$</td>
<td>$\tau_E = 2.5 \text{ sec}$</td>
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</table>
Summary of Plasma Requirements

- $T = 15 \text{ keV}$ The RF community
- $p = 7 \text{ atm}$ The MHD community
- $\tau_E = 1.2 \text{ sec}$ The transport community
Where do we stand now?

• Heating: Tokamaks have already achieved

\[ T \approx 30 \text{ keV} \]

• Should extrapolate to a fusion reactor
Where do we stand now?

• Pressure: Pressure is normally measured in terms of $\beta$

$$\beta = \frac{2\mu_0 p}{B_0^2} \quad B_0 = B_{\text{max}} \left(1 - \frac{a + b}{R_0}\right)$$

• Existing tokamaks have achieved $\beta \sim 10\%$ although at lower magnetic fields

• In a reactor $p = 7 \text{ atm}$ corresponds to $\beta \sim 8\%$
Where do we stand now?

• Energy confinement time: \( \tau_E \) is usually determined empirically

\[
\tau_E = 0.26 \frac{I_M^{1.06} R_0^{1.39} a^{0.58} n_{20}^{0.41}}{P_M^{0.69} B_0^{0.41}}
\]

• Experiments have achieved \( \tau_E \approx 0.3 \) sec at lower magnetic fields

• In a reactor \( \tau_E \approx 1 \) sec should be achievable (requires \( I_M = 17 \text{ MA} \))
Doesn’t this mean we have succeeded?

• No!!!

• There is one crucial unsolved reactor problem

• A power reactor should be a steady state device, not a pulsed device

• Simple tokamaks are inherently pulsed devices because of the OH transformer
Why are tokamaks pulsed devices?

- To hold the plasma in equilibrium an OH-PF field system is needed.
- The toroidal current is normally driven by a transformer which is inherently pulsed.

![Diagram showing OH Transformer and PF Coil]
How can we resolve this problem?

• Approach #1 Advanced tokamak operation

• Approach #2 The stellarator
The Advanced Tokamak

- The advanced tokamak achieves steady state by non-inductive current drive

- Directed RF waves trap and drag electrons with the wave generating a current
This works but...

- Current drive efficiency is low
  \[ P_{RF}(\text{watts}) \approx 10 \, I_{CD}(\text{amps}) \]

- For our reactor \( I = 17 \, \text{MA} \)

- With efficiencies this implies \( P_{RF} \approx 340 \, \text{MW} \)

- This is unacceptable from an economic point of view
Is there any way out?

- Possibly

- In a torus there is a naturally driven transport current

- It is known as the bootstrap current $J_B$

- No current drive is required

- If enough $J_B$ current flows (75%), the current drive requirements can be dramatically reduced
How much bootstrap current flows?

• The formula for the bootstrap fraction is

\[ f_B \equiv \frac{l_B}{l} \approx \frac{1}{3} \frac{\beta q_*^2}{\epsilon^{3/2}} \propto \frac{a^{5/2}}{R_0^{1/2}} \frac{p}{l^2} \]

\[ q_* = \frac{5 a^2 B_0}{R_0 l_M} \sim \frac{1}{l_M} \]

\[ \epsilon = \frac{a}{R_0} \]

• To make \( f_B = 75\% \) requires a combination of high pressure and low current
Are there limits on $p$ and $I$?

- Yes!!
- If they are violated a major disruption can occur
- A catastrophic collapse of the $p$ and $I$
- Major disruptions must be avoided in a reactor or ITER
Specific Plasma limitations

- $l_{max}$ is limited by kink stability condition
  \[ q_* \equiv \frac{5 a^2 \kappa B_0}{l_M} \geq 2 \]

- The elongation $\kappa$ is limited by vertical instabilities
  \[ \kappa < 2 \]

- But $l_{min}$ is limited by transport: $\tau_E \sim l_M$

- Beta is limited by the no wall Troyon limit
  \[ \beta < \beta_N \frac{l_M}{aB_0} \quad \beta_N \approx 0.03 \]

- This generates too low a bootstrap fraction
What do we do now?

- The best approach: Hollow $J$ profiles
  - Perfectly conducting wall
- Hollow $J$ means less $I_M$
- A conducting wall raises the $\beta$ limit by as much as a factor of 2
But the wall has a finite conductivity

- A finite $\sigma$ wall slows down $\gamma$, but leaves $\beta_{\text{crit}}$ the same as without a wall

- This slow growing mode is known as the resistive wall mode

- It is a major impediment to steady state operation
Can the resistive wall mode be stabilized?

- Feedback may work but is somewhat complicated

- Plasma flow can stabilize the mode but high flow velocities $v \sim v_{\text{thermal}}$ are needed and are difficult to initiate and maintain

- Plasma kinetic effects may also play an important role

- This is a crucial unresolved problem in tokamak research
Any other MHD reactor problems?

- Edge localized modes (ELMs)

- These are bursts of plasma energy from the plasma edge that occur when the pressure gets too high

- MHD modes driven by the edge $\nabla p$ and $J$

- The edge acts like a pressure relief valve

- This should be a good way to control and stabilize the edge plasma pressure
But

- There are several types of ELMs
- Most are bad
- Type I = bad
- Type II = good
- Type III = bad
- Difficult to predict which type ELMs will be present
- Another very important MHD problem
Any more problems?

- The neoclassical tearing mode (NTM)
- Resistive tearing mode including toroidal trapped particle effects
- Requires a finite seed island to grow (e.g. due to sawteeth)
- NTMs can be excited at lower $\beta$ than ideal MHD modes
- The $m=3/n=2$ mode can lead to enhanced transport
- The $m=2/n=1$ mode can lead to disruptions
Preventing NTMs

• Preventing NTMs #1: Reduce seed island by sawtooth destabilization (e.g. ICCD at the $q = 1$ surface)

• Preventing NTMs #2: Reduce island width by external control (e.g. ECCD at $q = 3/2$ surface)
Happy endings for AT problems?

• Stabilize the resistive wall mode
  (feedback, rotation, kinetic effects)

• Control ELMs (edge $\nabla p$ driven modes,
  affected by shear flow in the edge)

• Prevent neoclassical tearing modes
  (eliminate seed island, limit island growth)
What about the spherical tokamak (ST)?

• The ST is an ultra-tight aspect ratio tokamak

  • $\varepsilon = a/R_0 \rightarrow 1$

• Capable of high beta since $\beta_{\text{crit}} \propto a/R_0$

Source: Princeton Plasma Physics Laboratory.
But

- \( p \propto \beta (B_{\text{max}})^2 (1-\varepsilon)^2 \)
- Pressure is not that high because of 1/R effect
- Large \( I \) is required – tough bootstrap problem
- Central TF leg must be copper: low \( B_{\text{max}} \) (7.5T) because of joule losses
My not so happy conclusion

• ST is very interesting plasma physics experiment

• The ST does not solve any of the difficulties of the standard tokamak

• The ST probably generates more new problems than in the standard tokamak.
The Stellarator

• An inherently 3-D configuration (a toroidal-helix)

• An inherently steady state device – resolves the current drive problem

• No toroidal I is required – should greatly reduce the kink driven disruption problem
Are there any problems?

- Stellarators are much more complicated technologically
- The magnet design in particular is complex
- 3-D equilibrium with closed flux surfaces can be calculated but with difficulty
- 3-D stability can be calculated but also with difficulty
- Stellarators are very flexible before they are built – many options. But are inflexible once they are built
The LHD (Japan $1B)

- Continuous wound superconducting coil
- Engineering marvel
- Technology doesn’t extrapolate well into a reactor

Diagram of Large Helical Device (LHD) reactor removed for copyright reasons.
Types of new stellarators

• Stellarator geometries are based largely on reducing 3-D neoclassical transport losses
• This has lead to the concept of quasi-symmetry
• Use of modular coils for reactor viability
What is quasi symmetry?

• It has been shown that the guiding center particle drift off a flux surface depends only on $|\mathbf{B}|$ not $\mathbf{B}$

$$\mathbf{n} \cdot \mathbf{V}_{\text{GC}} = f(|\mathbf{B}|, \psi)$$
Three symmetries

- General stellarator field
  \[ \mathbf{B} = B_0 \frac{R_0}{R} \mathbf{e}_\phi + \sum b_{m,n}(r) \exp[i(m\theta + n\phi)] \]

- Quasi-poloidal symmetry (W7-X): \(|\mathbf{B}| \approx f(\phi, \psi)\)

- Quasi-toroidal symmetry (NCSX): \(|\mathbf{B}| \approx f(\theta, \psi)\)

- Quasi-helical symmetry (HSX): \(|\mathbf{B}| \approx f(M\theta + N\phi, \psi)\)
W7-X (Germany $1B)

- Modular superconducting coil for reactor viability
- Very low bootstrap current
- Large aspect ratio: $R_0/a = 10$

NCSX (USA $100M)

- Modular copper coils
- Significant bootstrap current
- Tight aspect ratio: \( R_0/a = 4 \)

Source: Princeton Plasma Physics Laboratory.
General behavior

• Confinement approaching that of a tokamak

• Beta limits not yet tested

• Heating seems to work but not yet at tokamak levels
A happy ending to stellarator problems?

• More efficient 3-D equilibrium codes

• More efficient 3-D stability codes

• Good coil design codes

• Less complicated, less expensive magnets
Summary of Talk

• We have accomplished a lot in MHD

• But there is still a lot more to do

• More in inventing new ideas than developing new tools
New Ideas Needed

- Stabilize the resistive wall mode
- Optimize the use of flow stabilization
- Predict and control ELMs
- Stabilize the neoclassical tearing mode
- Invent ever cleverer stellarator geometries
- Develop less expensive stellarator magnets
New Theory Tools Needed

• More efficient 3-D equilibrium codes
• More efficient 3-D stability codes
• Development of hybrid MHD-kinetic codes