Problem Set 2  
Due SES #6


Suggested Problems: [EL] Chapter 2, Problems 2.1, 2.3, 2.4, 2.11, 2.13

**Question 1:** Cs-137 is an important radioisotope used in the treatment of cancer. This isotope beta decays into a meta-stable form of Ba-137, releasing an electron with a kinetic energy of 1.176 MeV in the process. Ba-137m, in turn, releases a gamma ray with an energy of .662 MeV and becomes stable. The best way to produce Cs-137 is found to be from fission of U-235. Your job is to determine the maximum amount of Cs-137 produced from fission using a target of low enriched U and thermal neutron beam of intensity $10^{17}$ n/cm²sec.

a. Develop a pair of coupled differential equations you can use to determine the composition of U-235 and Cs-137 at any given time. Define any assumptions made or nifty approximations used. Assume the flux within the target is constant at the intensity stated above.

b. If the target contains 25 kg. of U-235 and the neutron flux within the target is constant and uniform (no neutron multiplication occurs), use the solution to the differential equations set up in part a. to determine the optimal time to withdraw the sample from the neutron beam and determine the maximum amount of Cs-137 produced. If the type of radiotherapy machine you’re making needs 50.9 TBq of Cs-137, how many radiotherapy machines can be produced using this maximum amount? Some useful information is provided in the box below.

<table>
<thead>
<tr>
<th>$\sigma_f^{U-235}$</th>
<th>100 b.</th>
<th>$\gamma_{Cs-137}$</th>
<th>6.337%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f^{U-235}$</td>
<td>585 b.</td>
<td>$t_{5}^{Cs-137}$</td>
<td>30.07 years</td>
</tr>
<tr>
<td>$\sigma_s^{U-235}$</td>
<td>15 b.</td>
<td>$\sigma_{\gamma}^{Cs-137}$</td>
<td>0.25 b.</td>
</tr>
</tbody>
</table>

The Cs-137 you produced is put into many radiation therapy machines which are then shipped off to hospitals all over the world. Some 20 years later, you receive a call from authorities in South America saying that some steel workers came across one of your machines in a junk yard and broke open the radioactive source.

c. Determine the activity of the source after a decay period of 20 years.

d. If all the energy from the beta particle and gamma ray from each decay is deposited in the body of one steel worker over an exposure period of 4 hours, determine the total energy deposited in his tissue. What is the rate of energy deposition [J/sec]?
**Question 2:** You wake up one day and find an article in the newspaper rambling on about proliferation worries spurted by the announced discovery of a natural reactor in North Korea, from which criticality was last achieved 2,000 years ago. The article maintains that enough Pu-239 was created such that the North Korean government could recover the Pu-239 produced in the natural reactor and create a nuclear bomb. You decide to check the numbers yourself and see if the source of this information actually knew what they were doing.

a. (5 pts.) State all the nuclear reactions and decay processes involved in the formation and decay of Pu-239 from U-238 in this natural reactor.

b. From the reactions and decay modes above, set up the full set of coupled differential equations by which you could determine the amount of Pu-239 formed over the lifetime of the reactor. Assume a time varying flux defined as follows to account for the change in moderation (water in the soil) over the course of a year.

\[
\phi(t) = \left(2 \times 10^9 \right) \sin(2 \pi t) + 2 \frac{n}{cm^2 \text{ sec}} \quad \text{where} \ t \ \text{is in years}
\]

c. What approximations can be made to simplify the equations formed in part b.? Write down the simplified set of differential equations.

d. If the original enrichment of the natural uranium was at 1% when the reactor started 12000 years ago, then use the simplified equations for U-235 and U-238 to determine the enrichment after fission stopped 2000 years ago.

e. Calculate the possible range of Pu-239 produced using the minimum and maximum flux values along with a steady state approximation for the differential equations formed in part c. Use the fact that the 2 metric ton deposit is made of 1% enriched uranium and assume the U-238 content is constant at this value for the calculations. Are the proliferation worries in the article sound or is the source of this information full of it? (The Nagasaki bomb used 6 kg. of Pu-239)

f. Why is the discovery of this natural reactor likely false?
