Problem Set 6
Due SES #18


Suggested Problems: [EL] Chapter 6, Problems 6.6, 6.12

Problem 1 [25 Points]

You wish to shield the neutrons produced from an inertial confinement fusion (ICF) device. The best shielding design you come up with is a spherical, annular shield of borated polyethylene in which the hydrogen will slow the 14.5 MeV neutrons down to thermal energies after which they will be absorbed by the boron present. The source at the center of the device will emit neutrons at $S_p$ neutrons per second. The inner radius of the shielding used is $R_i$ meters and the outer radius is $R_o$ meters. Assume there is no material between the neutron source and the shielding.

a) Use the diffusion equation for this 1 group, non-multiplying case to determine and plot the flux distribution throughout the shielding. Assume the shielding is uniform and has an absorption cross section of $\Sigma_a$ and a diffusion coefficient of $D$. (Note: This is slightly different from what was done in class, so before you throw any terms away, take a close look at the boundary conditions for this geometry)

b) Determine what fraction of the source will be reflected back from the inner surface towards the center of the ICF device. Important parameters are defined below. (Use the definition of albedo in Lewis, Chapter 6. Useful equations: 6.33 and 6.40)

c) A coolant system will need to be designed, though, because the absorption of neutrons will produce alpha particles which will locally deposit 2.82 MeV of energy in the shielding. Using the parameters given below, determine the amount of heat needed to be extracted from the shielding.

\[
S_p = 10^{20} \text{ n/sec}
\]
\[
R_i = 100 \text{ cm.}
\]
\[
R_o = 150 \text{ cm.}
\]
\[
D = 2/3 \text{ cm.}
\]
\[
\Sigma_a = 1 \text{ cm}^{-1}
\]

Problem 2 [25 Points]

A high energy proton beam is incident on a slab of graphite surrounded by a region of coolant. The interaction of the protons with carbon-12 produces a uniform neutron source throughout the graphite slab only. You can take both the graphite and surrounding coolant to be infinite slabs in the y and z direction. The graphite region has a thickness of a cm. and the surrounding coolant has a thickness of b cm. The volumetric neutron
source within the graphite slab is defined as \( S_o \), with units \([n/cm^3/sec]\).

a) Determine and plot the flux distribution throughout graphite slab with and without the coolant surrounding it. Assume \( D \) and \( \Sigma_a \) are the same for the graphite and the coolant.

b) Calculate the fraction of neutrons that will be re-emitted back towards the graphite slab from the edge of the slab for both cases evaluated in part a. Use the parameters defined below. What does this tell you about the effect the coolant has on the flux?

\[
\begin{align*}
\text{a}=10 \text{ cm.} \\
\text{b}=5 \text{ cm.} \\
\text{D}=1 \text{ cm.} \\
\Sigma_a=.1 \text{ cm}^{-1}
\end{align*}
\]

**Problem 3 [20 Points]**

In a homogeneous 1D infinite slab, a single planar source is placed at \( x=x_0 \) which gives off neutrons isotropically at a rate of \( S_o \ n/cm^2 \text{ sec} \).

a) Write down the correct diffusion equation for this problem.

b) Solve the diffusion equation using the appropriate boundary conditions in order to determine the flux distribution.

c) Instead of a single planar source, we instead distribute a source which varies which varies within the slab. Instead of solving the diffusion equation all over again, substitute the following relation in for \( S_o \) in the previous solution and integrate over the position of the source.

\[
S(x) = \begin{cases} 
S_o \cos\left(\frac{\pi x}{2A}\right) & \text{for } -A \leq x \leq A \\
0 & \text{for } |x| > A 
\end{cases}
\]

This will give you the flux at any position within the slab. Be careful with your limits of integration, there should be an absolute value in your integral! Plot the resulting flux distribution from -3A to 3A. (Note that \( S_o \) is in units of \( n/cm^3 \text{ sec} \) for this problem)

\[
\text{A}=5 \text{ cm.} \\
\text{S_o}=10^{14} \ n/cm^3 \text{ sec} \\
\text{D}=1 \text{ cm.} \\
\Sigma_a=.1 \text{ cm}^{-1}
\]

d) What expression does your answer to part c reduce down to when \( x \) gets further away from the source region?