1. Give the functional form and sketch both the real and imaginary components of the Fourier transformation (with respect to $x$) of the following functions. Identify the spatial frequencies, wave-numbers, and features in the spectra. $k_o$ is a constant real number

   a. $\cos(k_o x) + i \sin(k_o x)$  
   (5 points)

   b. $\text{TopHat}(\frac{x}{4})$  
   (5 Points)

   c. $\sin(8k_o x)\cos(k_o x)$  
   (5 points)

   d. $\sum_{n=-2}^{2} \delta(x-n) = \sum_{n=-\infty}^{\infty} \delta(x-n) \cdot \text{TopHat}(\frac{x}{2})$  
   (5 Points)

   e. $\text{TopHat}(x-4)$  
   (5 Points)

2. Show that the $k=0$ point of $F(k)$ is equal to the area $f(x)$, where $f(x) \leftrightarrow F(k)$.  
   (10 points)

3. Show that the $k_x = 0$ point of $F(k_x,k_y)$ is equal to the projection of $f(x,y)$ onto the y-axis where $f(x,y) \leftrightarrow F(k_x,k_y)$.  
   (5 points)

4. A simple way of characterizing the spatial distribution of a radiation source is to image it with a pin-hole imager.  
   (40 points)

   a. Draw the experimental geometry, and explain why this is a useful experiment. What is the form of the image, $I(x,y)$, in terms of the source distribution, $S(x,y)$, assuming a perfect pin-hole camera? Place the a distance, $a$, from the source and the screen (detectors) and a distance, $b$, from the pin-hole. Include the magnification in your answer.  
   (20 points)
b. Assume the pin-hole is of a finite size (radius = r), and therefore a true representation of the source is not observed. In linear imaging terms, explicitly describe the detected signal, $I(x,y)$ in both the source distribution and the pin-hole size.  

(10 points)

c. There is an oblique effect if the source extends far from the central line through the pin-hole. Without calculating the geometry of this effect, describe how it arises. 

(10 points)

5. The Nyquist condition states that to correctly measure a frequency the signal must be sampled at least twice a period.  

(20 points)

a. Let $f$ be the Nyquist frequency, show that the signals, 

$$\cos[2\pi(f + \Delta f)t] \text{ and } \cos[2\pi(f - \Delta f)t],$$

lead to the exact same data points when sampled at times $t(n) = \frac{n}{2f}$.  

(10 points)

b. Explain aliasing in terms of the above result.  

(10 points)