Problem Set No. 2

1. It is known that the proton and neutron in a deuteron can be broken apart by irradiating the deuteron with high-energy gamma rays. This is called the photo-nuclear reaction. It is found that the threshold energy of the photo-nuclear reaction is $E_\gamma = 2.225\text{MeV}$. Based on this energy and the known masses, $M_H = 1.0078252\text{amu}$ and $M_D = 2.0141022\text{amu}$, calculate the neutron mass.

2. Show by the theory of electrostatic that the repulsive energy due to a uniform distribution of charge $Ze$ in bulk of a sphere of radius $R$ is $E = \frac{3}{5} \frac{(Ze)^2}{R}$. Show that the coefficient in this expression will become $1/2$, if the charge is instead uniformly distributed over the surface. What would be the coefficient, if the charge is uniformly distributed over a spherical shell of an inner radius $R_1$ and an outer radius $R$?

3. The semi-empirical mass formula with the Coulomb term derived from the assumption of a homogeneous volume distribution of charge yields values for the mass which, in the region of heavy nuclei, differ from measured values by at most 0.01 percent. Using $^{238}\text{U}$ as an example ($M = 238.12\text{ amu}$), show that this error limit absolutely rules out the other alternative of a uniform surface distribution of charge, even if the mass discrepancy were due exclusively to the Coulomb term. One could nevertheless conceive of other forms of charge distribution; for instance, a homogeneous distribution throughout the volume of a hollow spherical shell having an inner radius $R_1$ and an outer radius $R$ (equal to the nuclear radius). What is the largest value of the ratio $R_1/R$ commensurate with the above mass?

4. Use the expression derived in class

$$Z_{\text{stable}} = -k_2/2k_3, \quad k_2 = -\left[4a_a + (M_N - M_H)\right], \quad k_3 = \frac{4a_a}{A} \left(1 + \frac{A^{2/3}}{4a_a / a_c}\right)$$

to establish whether $^{54}\text{Xe}^{142}$ is $\beta^-$ unstable or $\beta^+$ unstable.

5. Show that all stable nuclides with $A \geq 140$ are unstable with respect to emission of an alpha-particle ($^2\text{He}^4$). As an example, you can show that the binding energy of an alpha-particle in $^{92}\text{U}^{235}$ is negative and equal to $-4.64\text{ MeV}$. Do the same thing for $^{94}\text{Pu}^{239}$. 

6. When a thermal neutron (a neutron with practically zero kinetic energy in the energy scale we are considering in this problem) is captured in a nucleus with \( Z \) protons and \( N \) neutrons by a reaction:

\[
zX^A +_0 n^1 \rightarrow zX^{A+1} + Q_n
\]

the resultant Q-value is called the binding energy of the last neutron.

(A) Show that the Q-value can be calculated in terms of the binding energy by a formula:

\[
Q_n = B(A + 1, Z) - B(A, Z)
\]

In the case where \( ZX^A \) is a fissionable nucleus, the \( Q_n \) can also be considered as the excitation energy available for inducing fission reaction of the compound nucleus \( ZX^{A+1} \).

You can use the semi-empirical formula

\[
B(A, Z) = a_v A - a_c e^{-\frac{Z^2}{A^{1/3}}} - a_s A^{2/3} - a_a \frac{(A - 2Z)^2}{A} + \delta
\]

where

\[
\delta = a_p A^{-3/4} \quad \text{for an e-e nucleus}, \quad \delta = -a_p A^{-3/4} \quad \text{for an o-o nucleus}, \quad \delta = 0 \quad \text{for an e-o or o-e nucleus}
\]

The constants in the semi-empirical formula have values (in unit of MeV):

\[
a_v = 14.1, \quad a_c = 0.595, \quad a_s = 13.0, \quad a_a = 19.0, \quad a_p = 33.5.
\]

(B) Consider the case of thermal neutron induced fission reaction. Calculate the excitation energy available for the case of \( ^{235}\text{U} \) explicitly using the given information above.

(C) Do the same thing for the case of \( ^{238}\text{U} \) and show that the excitation energy available for this latter case is approximately 1 MeV less than the former case.

It is known that thermal neutrons can induce fission in \( ^{235}\text{U} \), but cannot do so in \( ^{238}\text{U} \). It is also known that in the latter case one needs to use high energy neutrons of energies greater than 1 MeV to induce the fission reaction. Your calculation is the explanation of this experimental fact.