(a) Plot reactivity, $\rho_\infty = \frac{k_\infty - 1}{k_\infty}$, vs. burnup for all cases and determine $B_1$ at $\rho_\infty = 0.03$ i.e. assuming leakage reactivity worth of 3% ;

(b) Qualitatively explain the relative behavior of the plots as determined by the relevant nuclear properties of the fissile and fertile species involved.

The initial reactivity is determined by the $\eta$-factor of the fissile nuclide as well as absorption cross-section of the fertile nuclide. U233 has higher $\eta$-factor value than other considered fissile nuclides and thus higher initial fuel reactivity despite the fact that Th is stronger thermal absorber than U238.

The slope of the reactivity versus burnup curve depends on fertile to fissile conversion efficiency as well as the $\eta$-factor of the fissile nuclide being bred. The conversion efficiency is primarily determined by the capture cross section of the fertile nuclide. This capture cross section is also somewhat affected by other nuclides present in the fuel. For example, the presence of strong thermal absorber would harden the spectrum and result in more efficient conversion.

A slight dip in reactivity at the beginning of U233–Th fuel irradiation is due to the captures in
intermediate nuclide – Pa233, which has relatively high absorption and long half life. In contrast, Np239 in the U238 – Pu239 chain does not affect reactivity significantly. In the U238 – Pu239 case, the reactivity drops faster for the initial 10-20 MWd/kg than thereafter. This is due to the fact that new Pu239 generated from U238 reaches equilibrium at about that burnup.

(c) For each fissile - fertile combination, determine the fissile enrichment (weight % of fissile nuclide in total HM) required to achieve 18 calendar months fuel cycle assuming a capacity factor of 90% and 3-batch fuel management.

Fuel Residence Time =
\[
\frac{365.25 \text{ days}}{12 \text{ month}} \times 18 \text{ month} \times 0.90 \frac{\text{Full Power Months}}{\text{Calendar Months}} \times 3 \text{ Cycles} \approx 1479 \text{ Effective Full Power Days}
\]

Dividing the unit cell power by the Heavy Metal mass (or directly from CASMO output)
Core average specific power = 34.67 W/g (U235-U238 fuel)
= 35.54 W/g (U233-Th232 fuel)
= 34.64 W/g (Pu239-U238 fuel)

Note that this approach neglects the presence of the guide tubes and inter-assembly coolant, which should be accounted for as core volume in the 104.5 W/cm³ power density value. Therefore, in practice, the specific power should be slightly higher than in the above example.

Required fuel discharge burnup = 34.67 × 1479 EFPD = 51.3 MWd/kg
= 35.54 × 1479 EFPD = 52.6 MWd/kg
= 34.64 × 1479 EFPD = 51.2 MWd/kg

Required \( B_1 = B_d \times \frac{N+1}{2N} = B_d \times \frac{2}{3} = \)
\[
= 51.3 \text{ MWd/kg} \times \frac{2}{3} = 34.2
\]
\[
= 52.6 \text{ MWd/kg} \times \frac{2}{3} = 35.0
\]
\[
= 51.2 \text{ MWd/kg} \times \frac{2}{3} = 34.2
\]

Running CASMO a number of times for various initial fissile nuclide densities, one could construct a relationship between enrichment and \( B_1 \) which turns out to be remarkably linear (at least in the practically significant \( B_1 \) values range). Using the above relationships we obtain 4.56, 3.77 and 4.97 weight % enrichment values for U235, U233, and Pu239 fuels respectively.

(d) Repeat the calculations of (c) for a 5-batch core.

For the 5-batch core with 18 months cycle

Fuel Residence Time =
\[
= \frac{365.25 \text{ days}}{12 \text{ month}} \times 18 \text{ month} \times 0.90 \frac{\text{Full Power Months}}{\text{Calendar Months}} \times 5 \text{ Cycles} \approx 2465 \text{ Effective Full Power Days}
\]
Required fuel discharge burnup \( = 34.67 \times 2465 \) EFPD = 85.5 MWd/kg
\( = 35.54 \times 2465 \) EFPD = 87.6 MWd/kg
\( = 34.64 \times 2465 \) EFPD = 85.4 MWd/kg

Required \( B_1 = B_d \times (N+1)/2N = B_d \times 6/10 \)
\( = 85.5 \) MWd/kg \( \times 0.6 = 51.3 \) MWd/kg
\( = 87.6 \) MWd/kg \( \times 0.6 = 52.6 \) MWd/kg
\( = 85.4 \) MWd/kg \( \times 0.6 = 51.2 \) MWd/kg

Using the same relationships between \( B_1 \) and initial enrichment, we obtain:
6.75, 5.19, and 7.35 weight % enrichment for U235, U233, and Pu239 fuels respectively.

(e) Assuming a 20% power uprate is desired, determine the required initial enrichment for each considered fissile-fertile combination if the cycle length is fixed to 18 months and the fuel is managed in 3-batches.

If the cycle length remains constant while power is increased by 20%, discharge burnup (as well as \( B_1 \)) should also increase by 20%.

Required \( B_1 = 34.2 \) MWd/kg \( \times 1.2 = 41.0 \) MWd/kg
\( 35.0 \) MWd/kg \( \times 1.2 = 42.1 \) MWd/kg
\( 34.2 \) MWd/kg \( \times 1.2 = 41.0 \) MWd/kg

Using the same relationships between \( B_1 \) and initial enrichment, we obtain:
5.43, 4.34, and 5.92 weight % enrichment for U235, U233, and Pu239 fuels respectively.

(f) If the coolant flow rate in this high power density core is also increased by 20%, what would be the implications to the MDNBR constraint? What would be the effect of this power uprate on reactivity control requirements?

CHF depends on local equilibrium quality, pressure, and flow rate for the fixed channel geometry. Since power to flow ratio is kept constant, the quality profile should remain the same. In case of uniform axial power shape, CHF varies linearly with the mass flux. However, 20% increase in the flow rate would result in only about 10% increase in CHF for typical PWR channel geometry and operating conditions (see Figure below) according to the standard W-3 correlation for CHF. In the up-rated core, the operational heat flux will increase by 20% whereas CHF increase will be smaller. Thus, MDNBR margin will be somewhat degraded.
Higher discharge burnup would necessitate higher initial enrichment and therefore higher initial reactivity of the fuel. This increase will have to be off set by burnable poisons and soluble boron. Higher initial enrichment will also slightly harden the neutron spectrum and thus reduce the worth of reactivity control materials.

An increase in the power density for a given assembly geometry will result in 20% increase in the linear heat generation rate, which will result in higher fuel temperature. Moreover, the heat transfer coefficient to the coolant will increase as a result of the flow rate increase but by less than 20% because of its non linear dependence on the flow rate, \( h \propto G^{0.8} \) (see Dittus-Boelter correlation). This will result in even higher fuel temperature. Therefore, the core reactivity will be reduced due to the larger Doppler Effect.

Further reduction in reactivity will be due to the higher equilibrium Xe concentration at higher power density.

Both of these effects will have to be compensated by even higher enrichment.

As mentioned earlier, since power to flow ratio is kept constant, the coolant temperature distribution and thus MTC will not be affected.