A typical PWR unit cell MCNP model is provided in /dsk2/users/251/MCNP.EXAMPLE/uo2.inp on mightyalpha.mit.edu machine. It models the geometry of representative PWR lattice cell with 4.5 w/o enriched fuel. Typical parameters are:

- Fuel (UO$_2$) Enrichment: 4.5 w/o
- Fuel (UO$_2$) Density: 10.4 g/cm$^3$
- Lattice Pin Pitch: 1.26 cm
- Fuel Temperature: 900 K
- Pellet Radius: 0.4096 cm
- Gap Thickness: 0.0082 cm
- Rod Diameter: 0.9500 cm
- Water Temperature: 583.1 K
- System Pressure: 15.5 MPa
- Power Density: 104.5 kW/liter-core

(a) Using the given MCNP model, run MCNP and calculate the following reaction rates (tally F4):  
- U-235 fission rate (use FM = −6)  
- U-238 capture rate (use FM = 102)

In addition, assume the two group model takes the boundary of 0.625 eV, and calculate the epithermal and thermal components of each reaction. Compute spectrum indices based on these reaction rates:

- Ratio of U-238 captures to U-235 fissions  
- Ratio of epithermal U-235 fissions to thermal U-235 fissions  
- Ratio of epithermal U-238 captures to thermal U-238 captures

(b) Calculate and plot the neutron spectrum inside the fuel pellet in 300 equal lethargy groups from 10 MeV to $10^{-3}$ eV. Harder spectrum can be achieved by either higher reload fuel enrichment ($X$) or a smaller hydrogen-to-heavy-metal (H/HM) ratio. Therefore, one can naturally give an asymptotic dependence of the epithermal-to-thermal flux ratio on the above two variables as:

$$\frac{\phi_2}{\phi_1} \approx \frac{H}{HM} \frac{1}{X}$$

Explain the physical meaning of this equation.
(c) Repeat the k-inf, spectrum and epithermal-to-thermal flux ratio calculations for the same fuel cell but with Uranium nitride (UN) fuel. Assume 14.3 g/cm³ density. Explain the differences in the results.

(d) For the UO₂ unit cell, modify the input file by adding a tally to obtain thermal, \( \phi_2 \), and epithermal flux, \( \phi_1 \), in the moderator. Calculate ratio of \( \phi_2/\phi_1 \) in the fuel and moderator. Discuss relative magnitude of the two and the reasons for differences.