Problem 1 (15%) – Sizing the shell of a spherical containment

i) The principal stresses for a thin spherical shell are:

\[
\sigma_r = \frac{-(p_i+p_o)}{2} \quad (1)
\]
\[
\sigma_\theta = \sigma_\phi = \frac{(p_i-p_o)R_c}{2t_c}
\]

where \(p_i=1.9 \text{ MPa}\) and \(p_o=0.1 \text{ MPa}\), \(R_c=12.5 \text{ m}\) and \(t_c\) is the shell thickness. Hook’s law yields:

\[
\varepsilon_\theta = \frac{u}{R_c} = \frac{1}{E} \left[ \sigma_\theta - \nu (\sigma_\phi + \sigma_r) \right] \quad (2)
\]

where \(E = 184 \text{ GPa}\) and \(\nu = 0.33\). Substituting Eq. (1) in Eq. (2), setting \(u=1 \text{ cm}\) and solving for \(t_c\), one gets:

\[
t_c = R_c \left( 1 - \nu \right) \frac{(p_i-p_o)}{[2E u/R_c - \nu (p_i+p_o)]} \approx 3.7 \text{ cm}
\]

Since \(R_c/t_c>10\), the thin shell assumption of accurate.

ii) The primary membrane general stress intensity for this case is:

\[
P_m = (\sigma_\theta - \sigma_r) \approx 102 \text{ MPa}
\]

\(\sigma_\theta\) and \(\sigma_r\) were calculated from Eq. (1) (thin shell assumption still applies), for \(t_c=8 \text{ cm}\). The ASME limit is \(S_m=110 \text{ MPa}\). Therefore the margin is \(S_m/P_m \approx 1.075\), or 7.5%.

Problem 2 (25%) – Reduction of containment pressure after LOCA

Conservation of energy for the containment:

\[
\frac{\partial E_{cv}}{\partial t} = \dot{Q}_{\text{decay}} - \dot{Q}_{\text{ex}} \quad (3)
\]

where \(\dot{Q}_{\text{decay}} = \dot{Q}_0 0.066t^{-0.2}\), \(\dot{Q}_0 = 1000 \text{ MW}\), and \(\dot{Q}_{\text{ex}} = 20 \text{ MW}\). Integrating Eq. (3):
\[ E_2 - E_1 = \dot{Q}_0 \frac{0.066}{0.8} t_2^{0.8} - \dot{Q}_{ss} t_2 \]  

(4)

Expanding the left-hand side, one gets:

\[
M_a c_{v,a}(T_2 - T_1) + M_w \{ [u_f(T_2)(1-x_2) + u_g(T_2)x_2] - [u_f(T_1)(1-x_1) + u_g(T_1)x_1] \} = \dot{Q}_0 \frac{0.066}{0.8} t_2^{0.8} - \dot{Q}_{ss} t_2
\]

(5)

where \(M_a, c_{v,a}, T_1, M_w\) and \(x_1\) are all known from the problem statement. The following equation holds for the control volume:

\[
V_{tot} = M_w [v_f(T_2)(1-x_2) + v_g(T_2)x_2]
\]

(6)

The containment pressure at \(t_2\), \(P_2=0.5 \text{ MPa}\), is the sum of the partial pressures of water and air:

\[
P_2 = P_{sat}(T_2) + \frac{M_a R T_2}{V_{tot} - M_w (1-x_2) v_f(T_2)}
\]

(7)

Therefore, Eqs. (5), (6) and (7) are 3 equations in the only unknowns \(t_2\), \(T_2\) and \(x_2\). Actually solving the equations, one finds \(t_2 \approx 14300 \text{ s}\), \(T_2 \approx 140.4 \degree \text{C}\) and \(x_2 \approx 0.035\).

**Problem 3 (45%) – Superheated Boiling Water Reactor**

i) T-s diagram:

![T-s Diagram](image)

ii) Taking the whole system as a control volume, the conservation of energy yields:

\[ 0 = \dot{Q} + \dot{m}_{FW} (h_{FW} - h_{sup}) \quad \Rightarrow \quad \dot{m}_{FW} = \dot{Q} / (h_{sup} - h_{FW}) \]

(8)
where $\dot{Q}=1000$ MW and $h_{FW}$ and $h_{sup}$ are the specific enthalpy of the feedwater and superheated steam, respectively. The difference $h_{sup}-h_{FW}$ can be expressed as follows:

$$
 h_{sup} - h_{FW} = c_{p,g} (T_{sup} - T_{sat}) + h_{fg} + c_{p,f} (T_{sat} - T_{FW}) \approx 2936 \text{ kJ/kg}
$$

where $T_{FW}=230^\circ\text{C}$ and $T_{sup}=510^\circ\text{C}$. Therefore, Eq. (8) yields $\dot{m}_{FW} \approx 340.6$ kg/s.

iii) The acceleration pressure drop is

$$
\Delta P_{acc} = G^2 \left[ \frac{1}{\rho^+_{m,\text{out}}} - \frac{1}{\rho^+_{m,\text{in}}} \right] 
$$

where $G = \dot{m}/A \approx 1800$ kg/m$^2$s, $\dot{m} = 2270$ kg/s, $A = 1.26$ m$^2$ and

$$
\rho^+_{m} \equiv \frac{1}{x^2 + \frac{(1-x)^2}{\alpha \rho_g (1-\alpha) \rho_f}}
$$

Since at the inlet there is only the liquid phase, it is $\rho^+_{m,\text{in}} = \rho_f$, while at the outlet $x=0.15$ and the void fraction can be found from the fundamental relation of two-phase flow:

$$
\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_f} \cdot \frac{1-x}{x} \cdot S} \approx 0.69
$$

where $S = 2$, per the problem statement. Then it is $\rho^+_{m,\text{out}} \approx 240.3$ kg/m$^3$ from Eq. (10), and finally Eq. (9) yields $\Delta P_{acc} \approx 9,235$ Pa

iv) Since the heat flux is axially constant, dryout would occur at the outlet (Point B). The critical quality at the outlet is found to be $x_{cr} \approx 0.344$ from the CISE-4 correlation with $L_b=3$ m, and the coefficients $a=0.5987$ and $b=2.2255$, calculated for $P=6$ MPa, $P_c=22.1$ MPa, $G=1800$ kg/m$^2$s>$G^* = 1211$ kg/m$^2$s, $D_c=0.02$ m.

Then the critical power of the $A \rightarrow B$ channels is $\dot{Q}_{cr,AB} = \dot{m} [c_{p,f} (T_{sat} - T_A) + x_{cr} h_{fg}] \approx 1311$ MW, where $T_A=268^\circ\text{C}$. So, the $CPR = \frac{\dot{Q}_{cr,AB}}{\dot{Q}_{AB}} \approx 2.12$, with $\dot{Q}_{AB} = \dot{m} [c_{p,f} (T_{sat} - T_A) + x_B h_{fg}] \approx 618$ MW being the operating power of the $A \rightarrow B$ channels, where $x_B=0.15$.

**Problem 4 (15%) – Thermodynamic analysis of a new power cycle**

To be thermodynamically feasible, the cycle must not violate the 1$^{st}$ and 2$^{nd}$ law of thermodynamics.

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Taking the whole power cycle as the control volume, the conservation of energy (1st law) becomes:

\[ 0 = \dot{Q} - \dot{W} + \dot{m}_{\text{sea}} (h_{\text{in}} - h_{\text{out}}) \]  \hspace{0.5cm} (11)

where steady-state was assumed and \( \dot{Q} = 1000 \text{ MW}, \dot{W} = 400 \text{ MW}, \dot{m}_{\text{sea}} = 15000 \text{ kg/s}, \)

\( (h_{\text{in}} - h_{\text{out}}) = c_{\text{sea}} (T_{\text{in}} - T_{\text{out}}) \), \( c_{\text{sea}} = 4000 \text{ J/kg°C} \) and \( T_{\text{in}} = 288 \text{ K (15°C)} \) and \( T_{\text{out}} = 298 \text{ K (25°C)} \).

Using these numbers, Eq. 11 is identically satisfied. Therefore, the cycle does not violate the 1st law.

With the same choice of control volume, the 2nd law becomes:

\[ 0 = \frac{\dot{Q}}{T_r} + \dot{m}_{\text{sea}} (s_{\text{in}} - s_{\text{out}}) + \dot{S}_{\text{gen}} \Rightarrow \dot{S}_{\text{gen}} = \dot{m}_{\text{sea}} (s_{\text{out}} - s_{\text{in}}) - \frac{\dot{Q}}{T_r} \]  \hspace{0.5cm} (12)

where \( T_r = 723 \text{ K (450°C)} \) and \( s_{\text{out}} - s_{\text{in}} = c_{\text{sea}} \ln \frac{T_{\text{out}}}{T_{\text{in}}} \). Then Eq. 12 yields \( \dot{S}_{\text{gen}} = 665 \text{ kW/K} > 0 \), therefore the cycle does not violate the 2nd law either.

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