Problem 1 (20%) – Calculation of Flow Quality from Void Fraction Measurements

The information missing is the mass flux (or a superficial velocity) in the downcomer. For example, if the mass flux were known, then the following set of equations would enable calculation of the flow quality:

\[
\alpha = \frac{1}{1 + \frac{\rho_v}{\rho_\ell} \cdot S \cdot \frac{1-x}{x}} \quad \text{(fundamental \(\alpha\)-x-S equation)}
\]

\[S = \frac{v_v}{v_\ell} \quad \text{(definition of slip ratio)}
\]

\[v_\ell - v_v = v_b \quad \text{(relative velocity; note that in general \(v_\ell > v_v\) in downflow)}
\]

\[G = \rho_v \alpha v_v + \rho_\ell (1-\alpha) v_\ell \quad \text{(mass flux)}
\]

The unknowns are \(x\), \(S\), \(v_v\), and \(v_\ell\).

Problem 2 (30%) – Pressure Drop in Accelerating Single-Phase Flow

i) For a perfectly incompressible fluid the density \(\rho\) is constant, and so the mass and momentum equations become, respectively:

\[
\frac{\partial G}{\partial z} = 0
\]

\[
\frac{\partial G}{\partial t} = -\frac{\partial P}{\partial z} - \frac{\partial}{\partial z} \left[ \frac{G^2}{\rho} \right] - \tau_w \frac{P_w}{A} - \rho g \cos \theta \quad \Rightarrow \quad \frac{\partial G}{\partial t} = -\frac{\partial P}{\partial z} - \frac{f}{D_e} \frac{G|G|}{2\rho} - \rho g
\]

where \(P_w\), \(A\) and \(D_e\) are the channel wetted perimeter, flow area and equivalent diameter, respectively.

ii) Integrating the momentum equation with respect to \(z\), one gets:

\[
P_{\text{inlet}} - P_{\text{outlet}} = \int_0^1 \frac{\partial G}{\partial t} dz + \int_0^1 \frac{f}{D_e} \frac{G|G|}{2\rho} dz + \int_0^1 \rho g dz
\]
The pressure at the inlet is constant by assumption. The first term on the right-hand side is also constant because \( G \) increases linearly with time. The third term on the right-hand side is constant because the fluid is incompressible. The second term on the right-hand side increases roughly as \( t^2 \). Therefore, the above equation suggests that the outlet pressure must decrease roughly as \( t^2 \).

**Problem 3 (50%) – Sizing of a Turbulent-Deposition Air/Water Separator**

i) The Ishii-Mishima correlation gives the value of the air superficial velocity at the onset of entrainment, \( j_v=15.7 \text{ m/s} \) (calculated with the thermophysical properties of Table 1). Thus the separator will have to operate at \( j_v=0.7 \times 15.7 \text{ m/s} \approx 11 \text{ m/s} \).

Then the diameter of the separator can be calculated from the following equation:

\[
j_v = \frac{xG}{\rho_v} = \frac{x\dot{m}}{\rho_v \left( \frac{\pi}{4} D^2 \right)} \Rightarrow D = \sqrt{\frac{4x\dot{m}}{\rho_v \pi j_v}} = 0.196 \text{ m}
\]

where \( x=0.95 \) and \( \dot{m} = 0.42 \text{ kg/s} \).

ii) A mass balance for the water droplets in the vapor core (see notes on annular flow) gives:

\[
\dot{m}(1-x) \frac{de}{dz} = -\pi D \Gamma_d
\]

where ‘e’ is the entrained liquid fraction \( (e=1 \text{ at the inlet}) \), and \( \Gamma_d \) is the rate of droplet deposition, which can be found as:

\[
\Gamma_d = K \frac{1-x}{x} \rho_v e
\]

where \( K=0.1 \text{ m/s} \) is the deposition coefficient given by the McCoy-Hanratty correlation. Integration of the mass balance equation gives:

\[
e(L) = e(0) \cdot \exp\left(-\frac{\pi DK \rho_v}{x\dot{m}} L\right)
\]

where \( L \) is the length of the separator. If ‘e’ is to decrease by 50%, then the required length is:

\[
L = \frac{x\dot{m}}{\pi DK \rho_v} \log(2) \approx 3.7 \text{ m}
\]

iii) The separation efficiency of the separator is 50%, since 50% of the initial moisture content is removed.