2.40) \[ P(1H) P(F/H) = .3 \cdot .25 = .075 \]
\[ P(2H) P(F/H) \cdot 2 = 2 \cdot .25 \cdot .05 = .025 \]
\[ P(F_{\text{snow}}) = .1 \]
\[ P(\text{Flood}) = .075 + .025 + .1 = .2 \Rightarrow 20\% \text{ chance of flood per year.} \]

2.43) \[ P(\text{detect diseased tree}) = P(A) + P(B\overbar{A}) = .8 + P(B/A)P(A) \]

a) \[ B \text{ and } A \text{ are independent} \quad \therefore \]
\[ P(\text{detect}) = .8 + (.9)(.2) = .98 \]

b) \[ P(\text{detected by 1 device}) = P(A \overbar{B}) + P(B\overbar{A}) \]
\[ = (.8)(.1) + (.9)(.2) = .26 \]

c) \[ P(\text{locating diseased tree}) = P(A_0)P(A_1)P(B) + P(B_0)P(B_1)P(A) + P(A_0B) \]
where \[ P(X_0) = \text{detector } X \text{ detected it} \quad P(X_1) = \text{detector } X \text{ located it} \]
\[ P(\text{locating}) = .8 \cdot .7 \cdot .1 + .2 \cdot .9 \cdot .1 + .8 \cdot .9 = .848 \]

3.1) \[ a) \quad x=0 \Rightarrow P(\overbar{A}\overbar{B}\overbar{C}) = (1-.05)(1-.8)(1-.2) = .08 \]
\[ x=1 \Rightarrow P(\overbar{A}\overbar{B}\overbar{C}) + P(A\overbar{B}\overbar{C}) + P(A\overbar{B}\overline{C}) = .42 \]
\[ x=2 \Rightarrow P(AB\overbar{C}) + P(A\overline{B}\overline{C}) + P(A\overline{B}\overline{C}) = .42 \]
\[ x=3 \Rightarrow P(ABC) = (.5)(.8)(.2) = .08 \]

b) \[ \text{PDF} \]
\[ P(x) = .42, .08 \]

\[ \text{CDF} \]
\[ \text{CDF graph with steps at } 0, 1, 2, 3, 4 \]
3.1 continue)  c) \( P(x \leq 2) = .42 + .42 + .08 = .92 \)

d) \( P(0 \leq x \leq 2) = .42 + .42 = .84 \)

3.12)  

a) i) mode: \( \bar{x} = \frac{300}{300} = 1 \) vehicles

ii) mean: \(<x> = \int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{300} f_1(x) dx + \int_{300}^{400} f_2(x) dx \)

where \( f_1(x) = \frac{P}{300} x \); \( f_2(x) = \frac{-P}{100} x + 4_p \)

where \( P \approx 200 + \frac{1}{2} (p)(100) = 1 \) due to normalization \( \Rightarrow P = \frac{1}{200} \)

\(<x> = \int_{0}^{300} (1.67 \times 10^{-5}) x^2 + \int_{300}^{400} (5 \times 10^{-5}) x^2 + .02 x \ dx
\]
\( = 150 - 617 + 700 = 233 \) cars

iii) median: \( F_x(x_{50}) = \int_{0}^{x_{50}} f_1(x) \ dx \) 

\( F_x(x_{50}) = .5 = \frac{1.67 \times 10^{-5}}{3} (x_{50}^3) \)

\( \Rightarrow x_{50} = 245 \) cars

iv) \( F_x(x_{90}) = .9 = \int_{0}^{400} f_1(x) dx + \int_{300}^{x} f_2(x) \ dx \)

\( \Rightarrow \frac{1}{2} (.5) - 2.5 \times 10^{-5} x^2 + 2 	imes 10^{-2} x - 3.9 = 0 \)

\( x_{90} = 337 \) cars

associated probabilities of exceedance = \( \int_{x}^{400} f(x) dx \)

i) \( \bar{x} = 300 \); \( P(\bar{x} \leq x) = 0.25 \)

ii) \( <x> = 233 \); \( P(x > 233) = 0.546 \)

iii) \( x_{50} = 245 \); \( \text{by definition } P(x > x_{50}) = .5 \)

iv) \( x_{90} = 337 \); \( \text{by definition } P(x > x_{90}) = .1 \)

b) \( P(\bar{x} \geq 300) = \frac{V_2(100)}{2} (\frac{1}{300}) = .25 \)

\( P(\bar{x} \geq 350) = \frac{V_2(50)}{2} (\frac{-350}{100 \cdot 200} + \frac{1}{50}) = .0625 \)

\( P(\text{exceed}) = .2(.25) + .8(.0625) = .1 \rightarrow 10% \)
3.14) a) \( \langle X \rangle = \sum_{i=1}^{7} x_i P(x_i) = 100,000 \left[ 5(5.5) + 6(0.3) + 7(1) + 7(1) \right] = \$570,000 \)

\[ \sigma^2 = \text{Var}(X) = \sum_{i=1}^{7} 100,000 \left[ (5-5.7)^2(0.5) + (5-5.7)^2(0.3) + 2(7-5.7)^2(0.1) \right] = \$78,000 \]

\[ \sigma = \sqrt{\sigma^2} = \sqrt{6.1 \times 10^9} = \$78,000 \]

3.19) known info: \( M = 30 \); \( P(40) = .9 \) \( \Rightarrow X_{40} = 40 \)

a) \( S = \frac{X - M}{\sigma} \Rightarrow .9 = \frac{40 - 30}{\sigma} \)

\[ \Rightarrow 1.285 = \frac{40 - 30}{\sigma} \Rightarrow \sigma = \frac{10}{1.285} = 7.78 \text{ days} \]

\[ P(X \leq 50) = \phi \left( \frac{50 - 30}{7.78} \right) = \phi(2.57) = 0.9949 \]

b) \( P(X \leq 6) = \phi \left( \frac{6 - 30}{7.78} \right) = \phi(-3.856) = 1 - \phi(3.856) = 1 - .999942 = .5 \times 10^{-5} \)

\[ \text{this probability is sufficiently small that a normal distribution can be considered a reasonable approximation.} \]

C) \( S = \frac{\ln X - \bar{x}}{S} \); \( \bar{x} = \ln 44 - \frac{1}{2} \bar{S} \)

\[ \bar{x}^2 = \ln \left( 1 + \frac{\sigma^2}{\mu^2} \right) \]

\[ \bar{x}^2 = \ln \left( 1 + \frac{1.285^2}{30} \right) = 6.5 \times 10^{-2} \Rightarrow \bar{x} = .255 \]

\[ \lambda = \ln M - \frac{1}{2} \bar{x}^2 = 3.369 \]

\[ S = \frac{\ln(50)-3.369}{.255} = 2.13 \Rightarrow \phi(2.13) = .983414 = P(X \leq 50) \]

3.22) Large normal distribution: \( M = 30 \); \( \text{COV} = \frac{\sigma}{\mu} = \frac{\sigma}{30} = 0.2 \)

\[ \bar{x}^2 = \ln \left( 1 + \frac{\sigma^2}{\mu^2} \right) = \ln \left( 1 + \frac{36}{3000} \right) = 0.0392 \Rightarrow \bar{x} = 0.198042 \]

\[ \lambda = \ln M - \frac{1}{2} \bar{x}^2 = 3.38159 \]

a) \( P(H_{\text{adequate}}) = P(H \geq 39) = 1 - \phi \left( \frac{\ln 39 - \bar{x}}{\bar{x}} \right) = 1 - \Phi(1.424) = 1 -.92 = 0.08 \)

b) \( P(44 \geq H \geq 39) = \phi \left( \frac{\ln 44 - \bar{x}}{\bar{x}} \right) - \phi \left( \frac{\ln 39 - \bar{x}}{\bar{x}} \right) = .058 \)

\[ P(H \leq 44|H \geq 39) = \frac{P(44 \geq H \geq 39)}{P(H > 39)} = \frac{.058}{.92} = 0.06 \]
3.26c) Note: there are several ways to do this problem.

1) plot the table and fit a curve to obtain a continuous distribution then do the calculations using numerical methods to integrate.

2) plot the table and notice that it closely resembles a normal distribution, and use the properties of a normal distribution (calculated μ and σ) to do the probability calculations. (can also use binomial dist. once you have Pₐ)

3) find the "fail" define a "failure" criterion and then find the probability of failure from the experimental data and apply treat them as Bernoulli trials using the binomial distribution to calculate the probabilities.

4) define a "failure" criterion and then find the failure rate from the experimental data and apply the Poisson distribution to find the probabilities.

In these solutions I will use method 3 to solve the problem:

(a) $\frac{\text{occurrences}}{20 \text{ years}} > 80 \Rightarrow P = \frac{3}{20} = 0.15$ \Rightarrow [\text{would also be } \lambda \text{ if }]

(b) in the next 10 years,

$$P(X=3) = \binom{10}{3}(0.15)^3 (1-0.15)^7 = 0.1298 \left[ \frac{n!}{x!(n-x)!} \right] \text{Binomial}$$

(c) $P (X \geq 1 \text{ for 3 years life}) = 1 - P(X=0) = 1 - \binom{3}{0}(1-0.15)^3 = 0.3859$

(d) if "failure" is $v=85 \text{ kph}$, then $P = \frac{1}{20} = 0.05$

$P (X \geq 1 \text{ for 3 years}) = 1 - P(X=0) = 1 - \binom{3}{0}(1-0.05)^3 = 0.1426$