1) Short Answer
   a) \( w^2_{pe} = 4 \pi N_e e^2 / m_e \); \( w^2_{pi} = 4 \pi N_i e^2 Z_i^2 / m_i \)
   b) \( w_{ci} = e Z_i B / m_i c \)
   c) \( k_e^2 = T_e / 4 \pi N_e e^2 \)  \( (\text{For} \ n_e Z_i = n_e \text{ Plasma}) \)
   d) TRUE;  e) TRUE;  f) FALSE;  g) TRUE;  h) TRUE
   i) FALSE

2) WAVE ENERGY & DISSIPATION
   Since \( \epsilon \) is analytic, it can be Taylor series expanded about \( \omega = \omega_r \) for small \( \delta / \omega_r \). (The derivative does not depend on direction in complex plane for an analytic function.)

\[
\epsilon(k, \omega_r + i \delta) \approx \epsilon_r(k, \omega_r) + i \frac{\delta \epsilon}{\delta \omega} + \frac{1}{2} \frac{\delta^2 \epsilon}{\delta \omega^2} \delta^2
\]

\( \rightarrow \) 2nd. Order (Tangent)
Evaluating REAL and IMAGINARY parts separately:

\[ \epsilon_r(k, \omega_r) = 0 \quad \text{GIVES} \quad \omega_r = \omega_r(k) \quad \text{DISSIPATION RELATION} \]

(Real frequency comes from reactive Response)

\[ \delta = -\frac{\epsilon_r(k, \omega_r)}{\partial \epsilon_r / \partial \omega_r} \]

\[ \frac{\delta}{\omega} = -\frac{\epsilon_r(k, \omega_r)}{W \partial \epsilon_r / \partial \omega} = -\frac{\text{DISSIPATION}}{\text{WAVE ENERGY}} \]

The growth (or damping) rate in dimensional terms is negative the ratio of DISSIPATION \( (\epsilon_r) \) to WAVE ENERGY \( (W \partial \epsilon_r / \partial \omega) \).

Positive dissipation implies damping (as positive energy modes) & negative dissipation leads to growth.

From Eq. (1)

\[ \epsilon(k, \omega_r) = 1 - \frac{w\omega_r^2}{k^2} \text{P} \int_{\omega} d\omega u - \frac{1}{\omega_r} \frac{dF}{k^2 du} \frac{\omega_r^2}{u = \omega_r / k} \]

\[ \epsilon_r = -\frac{1}{k^2 \partial u} \frac{\omega_r^2}{u = \omega_r / k} \]
3) **Electron Beam Instabilities**

Evaluate labels:

\[ V_e = V_0 = \sqrt{2eI/m_e} = \sqrt{\frac{2k_{eV}}{m_e c^2}} \quad c \]

\[ V_{b1} = \sqrt{20keV/m_e} \quad \text{and} \quad V_{b2} = \sqrt{200keV/m_e} \]

Waves are high phase velocity e-

PLASMA OSCILLATIONS with \( \omega_r = \omega_p e \)

(Given \( \omega / k >> V_e \), \( \omega / k >> V_{b1} / n \), \( n << 1 \) for beam)

There are 2 UNSTABLE BANDS where
distribution has positive slope:

**Band 1:** \( V_m < \frac{\omega}{k} < V_{b1} \); \( \frac{\omega}{k} >> V_e \)

**Band 2:** \( V_m < \frac{\omega}{k} < V_{b2} \); \( \frac{\omega}{k} >> V_{b1}, V_e \)

**Wave Energy**

\[ E_r = 1 - \frac{\omega p^2}{\omega^2} = \omega d \phi / d\omega = Z \text{ wave energy} \]

This can be argued on physical grounds

or shown via expansion:

\[ E_r = 1 - \frac{\omega p^2}{\omega^2} \int \frac{k}{k^2} \left( 1 - \frac{1}{k^2} \right) \frac{d\phi}{d\omega} = \int \frac{\omega p^2}{k^2} \left( 1 + \frac{\omega p^2}{k^2} \right) d\omega \]

\[ = 1 - \frac{\omega p^2}{\omega^2} + \text{both-Gross corrections go to zero} \]
The (weak) dissipation is simply

\[ \varepsilon_i = -\frac{\mathbf{w}_e^2}{k^2} \frac{\partial F}{\partial u} \bigg|_{u=\mathbf{w}_e/k} \]

\( F \) is a sum of Maxwellians:

\[ F = \frac{1}{\sqrt{\pi} \mathbf{v}_e} \left[ \left( 1 - \frac{n_0 + n_b}{n} \right) e^{-\frac{V_e^2}{2V_e^2}} + \frac{n_{b1} e^{-\frac{(V_e-V_{b1})^2}{2V_e^2}}}{n} \right. \]

\[ \left. + \frac{n_{b2} e^{-\frac{(V_e-V_{b2})^2}{2V_e^2}}}{n} \right] \]

Growth rates will be determined from \( \frac{dF}{du} \) at wave number, \( k \), giving resonant phase velocity: \( \mathbf{v}_p = \mathbf{w}_e/k \).

**Most Unstable Modes**

Here we need only find maximum in \( \frac{dF}{du} \) for each beam:

\[ \frac{d^2 F}{du^2} = \mathbf{g} = -\frac{V_e}{2V_e} \left( 1 - \frac{2(V_e-V_{b1})^2}{2V_e^2} \right) \frac{n_{b1} e^{-\frac{(V_e-V_{b1})^2}{2V_e^2}}}{n} \]

\[ \Rightarrow \mathbf{v}_p (\max \delta) = V_{b1} - \frac{V_e}{\sqrt{2}} \]

or

\[ \mathbf{k}_{\max} = \frac{\mathbf{w}_e}{V_{b1} - V_e \sqrt{2}} = \frac{\mathbf{w}_e}{V_{b1}} \]
\[ \frac{du}{dn} \bigg|_{\text{max}} = \sqrt{\frac{2}{\pi}} \frac{n_{b1}}{n v e^2} e^{-\frac{12}{12}} = \sqrt{\frac{2}{\pi e}} \frac{n_{b1}}{n v e^2} \]

Maximum growth rate in Band 1 is therefore:

\[ \frac{V_{\text{max}}}{\omega_p e^{-\frac{1}{2}}} = \frac{\sqrt{4 e}}{2 v e^2} \frac{n_{b1}}{k_{\text{max}}} e^{-\frac{1}{2}} \approx \sqrt{\frac{4 e}{\pi}} \frac{n_{b1}}{v e^2} \]

Similarly for Band 2 (1 + 2, same formula)

Since \( \omega_p e^{-\frac{1}{2}} \) the wave resonant at maximum gradient of highest energy beam will have highest growth rate. With \( \omega_p e^{-\frac{1}{2}} \) this mode will become dominant as \( \omega \) gets large (in the linear regime).

4) Ion Acoustic Waves

Limiting forms of response for: \( V_e < \frac{W}{k e} \) electrons

\[ \frac{\omega}{k v e} \frac{1}{2} \left( \frac{W}{k v e} \right) \rightarrow \frac{W}{k v e} \left( \frac{e}{k v e} \right)^2 \rightarrow 0 \]

\[ N_e = \frac{1}{k v e^2} \quad \text{adiabatic response} \]
\[ \lim_{s \to \infty} s = \frac{\omega}{k v_c} = 1 \]

\[ z(s) = \frac{1}{\sqrt{s}} \left[ 1 + e^{-t^2} \left( -\frac{1}{3} t^2 + \frac{t^4}{5} + \cdots \right) \right] \]

\[ \approx -\frac{1}{3} \left( 1 + \frac{1}{2} \delta^2 \right) \]

\[ \Rightarrow 1 + s^2 z(s) = -\frac{1}{2 \delta^2} = -\frac{k^2 v_c^2}{2 \omega^2} = -\frac{k^2 T_e}{\omega^2 m_i} \]

Combining back to give \( \varepsilon \):

\[ \varepsilon = 1 + \frac{1}{k^2 \lambda_e^2} + \frac{1}{k^2 \lambda_e^2} \left[ -\frac{k^2 T_e}{\omega^2 m_i} \right] \]

\[ = 1 + \frac{1}{k^2 \lambda_e^2} \left( 1 - \frac{k^2 \xi^2}{\omega^2} \right) \quad \xi^2 = T_e/m_i \]

For \( k^2 \lambda_i^2 < 1 \), we may neglect the "1" term in dielectric to give:

\[ \omega^2 = k^2 \xi^2 \]

Note that "1" corresponds to 1s of Poisson's equation and ignoring it implies \( \bar{\varepsilon} = \varepsilon_i \). This is quasi-neutrality.

It is a result of Debye shielding of ion fluctuations by electrons in the long wavelength, \( k \lambda_e \ll 1 \), limit.
5) **ION–ELECTRON COLLISIONS**

Since collisions conserve energy, we know that:

\[
\left( d^3v \left[ \frac{1}{2} m_e v^2 C_{ee} (f_e, f_i) + \frac{1}{2} m_i v^2 C_{i} (f_i, f_e) \right] \right) = 0
\]

The first term represents the rate of change of electron energy due to e– collisions with ions. The second term represents the rate of change of ion energy due to ion collisions with electrons. These two terms must be equal or opposite to

\[
\left( d^3v \frac{1}{2} m_i v^2 C_{i} (f_i, f_e) \right) = \frac{2 e}{m_i} \sqrt{\frac{2}{\pi}} N (T_e - T_i)
\]

by Maxwellians.

If \( T_e > T_i \), the ion are heated until \( T_i = T_e \) or equilibrium is achieved.
6) **Thermal Equilibrium**

a) **Method of Lagrange Multipliers**

Perform functional variation on

\[ L = S + \alpha f + \beta E \]  

with \( \alpha, \beta \) multipliers

\[ \delta L = \int d^3v \left[ \delta (f \delta f) + \alpha \delta f + \beta \frac{1}{2} \nu^2 \delta f \right] \]

\[ = \int d^3v \delta f \left[ \ln f + 1 + \alpha + \beta \frac{1}{2} \nu^2 \right] \]

\[ \delta L = 0 \quad \delta f = 0 \quad \ln f = -1 + \alpha + \beta \frac{1}{2} \nu^2 \]

\[ f = \exp \left( 1 - \alpha + \beta \frac{1}{2} \nu^2 \right) \]

\( \alpha \) and \( \beta \) can be calculated from \( n, E \)

Define \( E = \frac{3}{2} n T \Rightarrow f = \left( \frac{m}{4\pi \hbar^2} \right)^{3/2} \exp \left( -\frac{1}{2} \frac{\nu^2}{T} \right) \)

b) **Entropy Increases Monotonically in Time Until** \( f = f_{\text{Max}} \) **At Which Point Entropy is Maximum.**

c) \( C_{ei}(f_e) = 0 = \sqrt{\frac{1}{2N}} \left[ f_e + \frac{T_e}{m_e V} \ln \frac{1}{2V} \right] \]

\[ = \text{constant} = 0 \quad \text{since} \]

\[ f_e \ln \frac{1}{2V} \to 0 \quad \text{as} \quad V \to \infty \]
\[ 0 = f_e + \frac{T_e}{m_e} \frac{d}{dV} f_e \Rightarrow f_e = \text{const} \times \exp\left(-\frac{m_e V^2}{2T_e}\right) \]

The term \( \frac{d}{dV} f_e \) represents a friction in velocity space, tending to drag particles to zero velocity. The term \( \frac{1}{2V} \frac{d}{dV} f_e \) represents a diffusion process causing distribution to spread in velocity space.

The Maxwellian distribution is the shape that balances these processes.