**The Levelized Cost of Production and the Annual Carrying Charge Factor**

First, define levelized cash flows:

1. **Discrete cash flows**

Consider the non-uniform cash flow series:

We can define an 'equivalent levelized' cash flow, $A_L$, such that the uniform series PW is equal to the PW of the actual series:

$$\sum_{n=1}^{N} A_{L}(P/F,i,n) = \sum_{n=1}^{N} A_{n}(P/F,i,n)$$

$$A_L = \frac{\sum_{n=1}^{N} A_{n}(P/F,i,n)}{\sum_{n=1}^{N} (P/F,i,n)}$$
2. Continuous cash flow rate

\[ A(t) \]

We obtain, by analogy,

\[
\bar{A}_L = \frac{\int_0^T A_0 e^{yt} dt}{\int_0^T e^{yt} dt}
\]

For the special case of an exponential increase in \( \bar{A} \)

\[
\bar{A}(t) = A_0 e^{yt}
\]

And expanding the exponentials as Taylor series and retaining terms through second order, yielding, to first order,

\[
\bar{A}_L = \frac{A_0}{r} \left[ 1 + \frac{r}{y} \left( 1 - e^{rt} \right) \right]
\]
Levelized Unit Cost of Product

The lifetime levelized cost, the constant cost that is equivalent in a present worth sense to the relevant time-varying cost, is a useful benchmark for comparisons of facilities which might otherwise be difficult to compare (e.g., windmills versus gas turbines.)

Example – manufacturing facility

Consider a factory with initial investment cost \( I_0 \) at \( t=0 \), which operates for \( N \) years after which it is salvaged at \( I_N \).

Suppose that during this period the factory produces \( Q_j \) units per year at an annual operating cost of \( M_j \) dollars per year.

What is the levelized cost of a unit of product – i.e., the uniform cost which, if recovered on every unit produced, will provide lifetime revenues just sufficient to cover all capital and operating costs?

Case I: No Taxes

Write the levelized unit cost, \( c \), as the sum of operating and capital components:
\[ c = c_m + c_l \]

1. Operating cost component, \( c_m \)

\[ \sum_{j=1}^{N} c_m Q_j(P/F,i,j) = \sum_{j=1}^{N} M_j(P/F,i,j) \]

\[ c_m = \frac{\sum_{j=1}^{N} M_j(P/F,i,j)}{\sum_{j=1}^{N} Q_j(P/F,i,j)} \]

2. Capital cost component, \( c_l \)
\[ \sum_{j=1}^{N} l_n (1+i)^{-j} = \sum_{j=1}^{N} c_i Q_j (P/F, i, j) \quad (1) \]

Define: Average (levelized) production rate \( Q_L \)

\[ \sum_{j=1}^{N} Q_L (P/F, i, j) = \sum_{j=1}^{N} Q_j (P/F, i, j) \]

\[ Q_L = \frac{\sum_{j=1}^{N} Q_j (P/F, i, j)}{(P/A, i, N)} \]

and substituting for \( Q_L \) in (1)

\[ c_i = \frac{1}{Q_L (P/A, i, N)} \left[ \sum_{j=1}^{N} l_n (P/F, i, N) \right] \]

\[ = \frac{1}{Q_L} \left[ l_0 (A/P, i, N) \sum_{j=1}^{N} l_n (A/F, i, N) \right] \]

i.e.,

levelized unit cost = \( \frac{1}{\text{levelized production rate}} \left[ l_0 \text{ capital recovery factor} \sum_{j=1}^{N} l_n \text{ sinking fund fac} \right] \)

Case II: With Taxes

\[ Q_c \]

\[ T_j \]

\[ M_j \]
As before, write $c = c_m + c_l$

Next, transform the cash flow problem into an equivalent tax-implicit problem

\[ Q_j(c_l + c_M)(1 - \theta) \]

\[ [D_j] \]

\[ I_N \]

\[ M_j(1 - \theta) \]

\[ I_o \]

And, decomposing into capital and operating components,

\[ C_i Q_j(1 - \theta) \]

\[ [D_j] \]

\[ I_N \]

\[ c_M Q_j(1 - \theta) \]

\[ M_j(1 - \theta) \]

\[ I_o \]

Then solve separately for $c_l$ and $c_M$. 
a. \( c_M \)

\[
(1 - \sum_{j=1}^{N} c_M Q_j(P/F,x,j)) = (1 - \sum_{j=1}^{N} M_j(P/F,x,j))
\]

\[
c_M = \frac{\sum_{j=1}^{N} M_j(P/F,x,j)}{\sum_{j=1}^{N} Q_j(P/F,x,j)}
\]

b. \( c_I \)

\[
(1 - \sum_{j=1}^{N} c_I Q_j(P/F,x,j)) = I_o - I_N \frac{(I_o - I_N)(P/A,x,N)}{N - D_j(j-1)}
\]

For the case of straight line depreciation:

\[
D_j = \frac{I_o - I_N}{N}
\]

and

\[
c_i = \frac{1 - \sum_{j=1}^{N} \frac{(I_o - I_N)(P/A,x,N)}{N - D_j(j-1)}}{\sum_{j=1}^{N} Q_j(P/F,x,j)}
\]

as before, define a levelized production rate, \( Q_L \)

\[
Q_L = \frac{\sum_{j=1}^{N} Q_j(P/F,x,j)}{\sum_{j=1}^{N} (P/F,x,j)} = \frac{\sum_{j=1}^{N} Q_j(P/F,x,j)}{(P/A,x,N)}
\]

And substituting in (2) above
\[ c_i = \frac{1}{(A/P, x, N) I_o (A/F, x, N) N I_o / I_o} \]

\[ = \frac{I_o}{Q_L} \frac{1}{(A/P, x, N) N I_o / I_o (A/F, x, N)} \]  

\[ (A/P, x, N) = \frac{x(1+x)^N}{(1+x)^N x} x \]

\[ (A/F, x, N) = \frac{x}{(1+x)^N x} 0 \]

\[ = \frac{x}{1} \]

This is a good approximation for large \( N \).

Notes

1. \( I_o \) is the PW of the initial investment at the start of operation.

2. In a tax-free environment \((t=0)\), the annual carrying charge factor reduces to the capital recovery factor, adjusted for NSV.

3. In the limit of large \( N \) \((N \rightarrow \infty)\)

4. The form of the annual capital charge factor in equation (3) applies to the case of straight-line depreciation. Equivalent expressions can be derived for other depreciation schedules.