1. **Bead on a Rotating Hoop**

   A bead lies on a frictionless hoop of radius $R$ that rotates around a vertical diameter with constant angular speed $\omega$, as shown in the figure below.

   ![Diagram](image)

   (a) What should $\omega$ be so that the bead maintains the same position on the hoop, at an angle $\theta$ with respect to the vertical? Express you answer in terms of some or all of the following: $\theta$, $R$ and $g$.

   (b) Analyzing the answer for Part A, you will find that there is a range of angular speeds, $0 < \omega < \omega_o$ for which the fixed angle $\theta = 0$ (meaning that the only balanced position is at the bottom of the hoop). Find the value of $\omega_o$. Express you answer in terms of some or all of the following: $R$ and $g$. 
2. Banked Turn

A car of mass \( m \) is going around a circular turn of radius \( R \), which is banked at an angle \( \beta \) with respect to the ground. Assume there is friction between the wheels and the road. Let \( \mu_s \) be the coefficient of static friction and \( g \) the magnitude of the gravitational acceleration. You may neglect kinetic friction (that is, the car’s tires do not slip). Derive an expression for the range of possible speeds \( v_{\text{min}} \leq v \leq v_{\text{max}} \) necessary to keep the car moving in a circle without slipping up or down the embanked turn. Express your answer in terms of some or all of the following: \( \mu_s, \beta, m, R \) and \( g \).
3. Tetherball Breaking Off

A small ball of mass $m$ is suspended by a string of length $l$. The string makes an angle $\beta$ with the vertical. The ball revolves in a circle with an unknown constant angular speed $\omega$. The orbital plane of the ball is at a height $h$ above the ground. Let $g$ be the gravitational constant. You may ignore air resistance and the size of the ball.

(a) Find an expression for the angular speed $\omega$. Express your answer in terms of some or all of the following: $l$, $\beta$, and $g$.

(b) Later, the ball detaches from the string just as it passes the $x$-axis. It flies through the air and hits the ground at an unknown horizontal distance $d$ from the point at which it detached from the string.

What horizontal distance $d$ does the ball traverse before it hits the ground? Express your answer in terms of some or all of the following: $l$, $\beta$ and $h$. 
4. Two Boxes Around a Shaft

Box 1 and box 2 are whirling around a shaft with a constant angular velocity of magnitude $\omega$. Box 1 is at a distance $d$ from the central axis, and box 2 is at a distance $2d$ from the axis. You may ignore the mass of the strings and neglect the effect of gravity. Express your answer in terms of $d$, $\omega$, $m_1$ and $m_2$, the masses of box 1 and 2.

(a) Calculate $T_B$, the tension in string B (the string connecting box 1 and box 2):
(b) Calculate $T_A$, the tension in string A (the string connecting box 1 and the shaft):
5. Satellite

(a) Two satellites are orbiting earth at different altitudes. Which satellite orbits at a higher speed \( v \) around earth? Assume that the orbits are circular and both satellites have the same mass.

(b) Which satellite orbits with a longer period, \( T \), around earth? Assume that the orbits are circular and both satellites have the same mass.
6. A coin on a rotating disk

A coin of mass \( m \) is on a rigid disk at a distance \( d \) from the center of the disk. There is friction between the coin and the disk. The coefficient of static friction is \( \mu_s \). At time \( t = 0 \), the disk begins to rotate with a constant angular acceleration of magnitude \( \alpha \). The magnitude of the acceleration due to gravity is \( g \).

Express your answers in terms of some or all of the given variables \( m, d, \mu_s, \alpha, t \) and \( g \) as needed.

(a) While the coin remains at rest relative to the disk, what is \( f_s \), the magnitude of the force of static friction exerted by the disk on the coin as a function of time \( t \)?

(b) At what angular speed \( \omega \) will the coin start to slip with respect to the disk?
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