10.4 System of Particles

Suppose we have a system of \( N \) particles labeled by the index \( i = 1, 2, 3, \ldots, N \). The force on the \( i \)th particle is

\[
\mathbf{F}_i = \mathbf{F}^\text{ext}_i + \sum_{j=1, j \neq i}^{i-1} \mathbf{F}_{i,j}. \tag{10.4.1}
\]

In this expression \( \mathbf{F}_{j,i} \) is the force on the \( i \)th particle due to the interaction between the \( i \)th and \( j \)th particles. We sum over all \( j \) particles with \( j \neq i \) since a particle cannot exert a force on itself (equivalently, we could define \( \mathbf{F}_{i,i} = \mathbf{0} \)), yielding the internal force acting on the \( i \)th particle,

\[
\mathbf{F}^\text{int}_i = \sum_{j=1, j \neq i}^{i-1} \mathbf{F}_{j,i}. \tag{10.4.2}
\]

The force acting on the system is the sum over all \( i \) particles of the force acting on each particle,

\[
\mathbf{F} = \sum_{i=1}^{i=N} \mathbf{F}_i = \sum_{i=1}^{i=N} \mathbf{F}^\text{ext}_i + \sum_{i=1}^{i=N} \sum_{j=1, j \neq i}^{j=N} \mathbf{F}_{j,i} = \mathbf{F}^\text{ext}. \tag{10.4.3}
\]

Note that the double sum vanishes,

\[
\sum_{i=1}^{i=N} \sum_{j=1, j \neq i}^{j=N} \mathbf{F}_{j,i} = \mathbf{0}, \tag{10.4.4}
\]

because all internal forces cancel in pairs,

\[
\mathbf{F}_{j,i} + \mathbf{F}_{i,j} = \mathbf{0}. \tag{10.4.5}
\]

The force on the \( i \)th particle is equal to the rate of change in momentum of the \( i \)th particle,

\[
\mathbf{F}_i = \frac{d\mathbf{p}_i}{dt}. \tag{10.4.6}
\]

When can now substitute Equation (10.4.6) into Equation (10.4.3) and determine that the external force is equal to the sum over all particles of the momentum change of each particle,

\[
\mathbf{F}^\text{ext} = \sum_{i=1}^{i=N} \frac{d\mathbf{p}_i}{dt}. \tag{10.4.7}
\]
The momentum of the system is given by the sum
\[
\vec{p}_{\text{sys}} = \sum_{i=1}^{N} \vec{p}_i,
\] (10.4.8)
momenta add as vectors. We conclude that the external force causes the momentum of
the system to change, and we thus restate and generalize Newton’s Second Law for a
system of objects as
\[
\vec{F}^{\text{ext}} = \frac{d\vec{p}_{\text{sys}}}{dt}.
\] (10.4.9)
In terms of impulse, this becomes the statement
\[
\Delta\vec{p}_{\text{sys}} = \int_{t_0}^{t_f} \vec{F}^{\text{ext}} \, dt \equiv \vec{I}.
\] (10.4.10)