Now let's go back and analyze that same problem that we were looking at before of a wheel rolling down an incline plane.

And it's rolling without slipping.

But instead of using the energy method, we're now going to use the torque method.

So let's consider the center of mass.

And what we want to do is draw the forces.

We have a normal force.

We have gravity.

And we have the friction force about the center of mass.

Our wheel has a radius R.

Let's choose a coordinate system i hat, j hat.

And so we have a right-handed system k hat, and that will correspond to some angle theta.

Now here we're now going to enlarge how we apply both translation and rotation.

And the beauty of this problem is we now can decompose our motion into translational motion and rotational motion.

So for the transitional motion, we'll apply Newton's second law.

Now, if this is the angle phi, then that's the angle phi as well.

And so our forces in the i hat direction, we have mg sine phi minus the friction force.

And that's equal to the x component of the acceleration.

Now we also can choose the center of mass to calculate the torque.

And so what we're really just studying here is simply our old problem in the center of mass frame of fixed axis rotation.

And you can see gravity is acting at the center of mass.
So it produces no torque about the center of mass.

The normal force is directed towards the center of mass.

And so when we take that vector product of \( R \) cross \( n \) from \( cm \) to this point down here, the contact point, these forces are anti-parallel.

So the normal force produces no torque.

And the only torque that we have is from the friction force, and that friction torque is going to give us a positive angular acceleration in the \( k \) hat direction.

It's at right angles with the vector \( R \).

So we have \( fs \) times \( R \) equals \( I \) center of mass times \( \alpha \).

And these are our two dynamic equations.

But remember, when the object is rolling without slipping, let's just remind ourselves that \( V_{cm} \) equals \( R \omega \).

And if I differentiate, the \( A_{cm} \) which is what we're calling \( ax \), is equal to \( R \alpha \).

So this \( ax \) here is the acceleration of the center of mass.

And that's our third condition.

And so now I see that I have three equations and my unknowns-- \( fs \), \( ax \) and \( \alpha \).

And so I'm going to solve these equations for \( a \).

And I'll look at these equations.

And what I'll do is I'll just substitute for \( \alpha \) \( ax \) over \( R \).

And then solve this equation for \( fs \), and put it in there.

And so I get \( mg \) sine \( \phi \).

Now my \( fs \) is equal to \( I_{cm} \) over \( R \) times \( \alpha \).

But \( \alpha \) is \( ax \) over \( R \). So that's \( ax \) and an \( R \) squared.

Notice dimensionally, \( I \) is \( mr^2 \).
So this is just ma, the dimensions of force, mg dimensions of force, and that's equal to max.

And now I can solve for ax.

And I get mg sine phi divided by m plus Icm over R squared.

Now ax is a constant.

And we can, from our kinematic equations, if our object is moving a distance s as it drops height h, we know from kinematics and we can work this out.

We have that Xcm is one half Acm t squared.

And we know that the velocity Vcm equals Acm times t.

And so when we put these two together, this is the distance s, we get that the velocity cm is equal to the square root of 2s times Acm.

Now s is equal to h over sine phi.

S h over s is sine phi.

And so the Vcm equals the square root of 2h sine phi times Acm-- but we've solved for Acm-- mg sine phi over m plus Icm over R squared.

And so we get the square root of 2mgh over m plus Icm divided by R squared.

And this agrees with our energy method.