Collaboration policy: You are encouraged to freely discuss homework problems with other 8.012 students and teaching staff. However, you must write up your solutions completely on your own—do not simply copy solutions from other students. You are forbidden from consulting solutions from previous years or from the web. Violations of this policy may result in disciplinary action.

Reading: Kleppner & Kolenkow, Chapters 1 and 2

0. Collaboration and discussion. Please list the names of all the students with whom you discussed these homework problems. Also be sure to write down your name and recitation section clearly on the first page.

1. Kleppner & Kolenkow, Problem 1.2 (5 points)

2. Kleppner & Kolenkow, Problem 1.7 (10 points)

3. Kleppner & Kolenkow, Problem 1.12 (10 points)

4. Kleppner & Kolenkow, Problem 1.13 (10 points)

5. Kleppner & Kolenkow, Problem 1.14 (5 points)

6. Kleppner & Kolenkow, Problem 1.15 (10 points)

7. Kleppner & Kolenkow, Problem 1.18 (10 points)

8. Kleppner & Kolenkow, Problem 1.20 (10 points)
   Note that “radial” and “tangential” accelerations correspond to the \( \hat{r} \) and \( \hat{\theta} \) components of \( \vec{a} \).

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9. **Two Trains and a Bee.** (15 points)

Consider two trains moving in opposite directions on the same track. The trains start simultaneously from two towns, Aville and Bville, separated by a distance $d$. Each train travels toward each other with constant speed $v$. A bee is initially located in front of the train in Aville. As the train departs Aville, the bee travels with speed $u > v$ along the track towards Bville. When it encounters the second train, it instantaneously reverses direction until it encounters the first train, then it reverses again, etc. The bee continues flying between the two trains until it is crushed between the trains impacting each other. The purpose of this problem is to compute the total distance flown by the bee until it is crushed. Assume that the bee is faster than the trains.

There are at least two good ways to solve this problem. One is to compute the distance for each flight leg and sum the resulting series. There is also another way to solve the problem with very little calculation. You are to do it both ways.

(a) (10 points) Find an expression for the distance $d_n$ covered by the bee after its $n^{th}$ encounter with a train. Define $d_0$ as the distance traveled during the first flight from Aville towards the train near Bville, $d_1$ the distance traveled by the bee during the first trip from the Bville train to the Aville train, etc. Sum the resulting series to get the final answer.

(b) (5 points) Devise another way to obtain the same answer using very little calculation.

10. **Dimensional Analysis.** (15 points)

You can often derive the solution of a problem by considering the dimensions of the relevant variables, including all related fundamental constants, and match the dimensions of the quantity you want to determine. This is called *dimensional analysis*, and it is a powerful approximation technique as well as a method of determining how quantities scale with different variables. The basic idea is to write the unknown quantity $(X)$ as a factor of all your relevant variables $(V_n)$ and constants $(C_n)$:

$$X = V_1^a V_2^b V_3^c C_1^d C_2^e \ldots$$

and then solve for the powers $a, b, c, d, e, \ldots$ so that the dimensions on both sides of the equation work out.

(a) (3 points) Derive an expression for the vibration frequency of a star of mass $M$ and radius $R$, if that vibration is caused by gravitational instabilities.

(b) (4 points) Derive an expression for the drag force on a ball of radius $R$ and mass $M$ moving with velocity $v$ through a medium with mass density $\rho$.

(c) (4 points) Derive an expression for the terminal velocity of a falling ball of radius $R$ and mass $M$ close to the surface of the Earth, when it experiences a drag force of the form $F = bv^2$. Can you find an alternate way of deriving this velocity?

(d) (4 points) Derive an expression for the frequency of a pendulum of mass $M$, hanging from a rope of length $L$ near the surface of the Earth, released from rest at an initial angle $\theta_0$. Warning! $\theta_0$ is dimensionless. Is it possible to constrain how the frequency depends on this variable?