Important Reminders

- Pset #8 due tomorrow.
- Problem Solving session in class tomorrow.
- MasteringPhysics due next Monday.
- **NOTE:** Class grading guidelines clearly allow discussion of MasterPhysics problems but also clearly prohibit directly working together or copying the answers of others.
- No Pset due next week.
- No formal tutoring sessions next week.

Center of Mass Velocity

- **Definition:** \( \vec{v}_{C.M.} = \frac{1}{M_{TOT}} \sum m_i \vec{v}_i \)
- **Connection to momentum:** \( M_{TOT} \vec{v}_{C.M.} = \vec{p}_{TOT} \)
- So, if momentum is conserved, the velocity of the center of mass is constant.

Simple Harmonic Motion - I

- **Start with Force equation:** \( \vec{F}_{Spring} = -k(\vec{l} - \vec{l}_0) \)
- Define x axis along direction spring is stretched and put \( x = 0 \) at the point the spring is unstretched:
  \[
  F_x = -kx = ma_x = m \left( \frac{d^2x}{dt^2} \right) \]
  \[
  \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m} x
  \]
- So, what is the solution to the differential equation?
Simple Harmonic Motion - II

The answer is sine and/or cosine function with three mathematically equivalent ways to write it:

\[ x = A \cos(\omega t + \phi) \]
\[ x = A \sin(\omega t + \phi) \]
\[ x = A \cos(\omega t) + B \sin(\omega t) \]

In all cases, \( A, B, \) and \( \phi \) are constants determined by the initial conditions.

\( \omega \) is given by the physics:

\[ \omega = \sqrt{\frac{k}{m}} \]

Simple Harmonic Motion - III

Connections to the physical motion:

\( A \) is the amplitude, the maximum displacement from zero

\( \phi \) is an arbitrary constant that depends only on when you define \( t = 0 \), no real connection to the nature of the motion

\( \omega \) is angular frequency in radians per second:

Frequency in cycles/second

\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

Period (time for one cycle)

\[ T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \]

Velocity/Acceleration in SHM

Also sine/cosine functions:

\[ x = A \cos(\omega t + \phi) \]
\[ v_x = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi) \]
\[ a_x = \frac{dv_x}{dt} = -A\omega^2 \cos(\omega t + \phi) \]
\[ = -\omega^2 x = -\frac{k}{m} x \]

Note that: Maximum speed \( |v_{\text{Max}}| = A\omega \)

Energy in SHM

Spring PE:

\[ PE_{\text{spring}} = \frac{1}{2} kx^2 = \frac{kA^2}{2} \left( \cos(\omega t + \phi) \right)^2 \]

Kinetic energy:

\[ KE = \frac{1}{2}mv^2 = \frac{m(A\omega)^2}{2} \left( \sin(\omega t + \phi) \right)^2 \]

Total:

\[ E_{\text{total}} = KE + PE = \frac{kA^2}{2} \left( \cos(\omega t + \phi) \right)^2 + \frac{m(A\omega)^2}{2} \left( \sin(\omega t + \phi) \right)^2 \]

\[ \omega^2 = \frac{k}{m} \]

\[ E_{\text{total}} = \frac{kA^2}{2} \left( \cos(\omega t + \phi) \right)^2 + \frac{1}{2} \left( \sin(\omega t + \phi) \right)^2 \]

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