Happy Holidays! Happy New Year! Welcome back!

- Today
  - Intro to angular motion

Important Concepts
- Equations for angular motion are mostly identical to those for linear motion with the names of the variables changed.
- Location where forces are applied is now important.

Important Reminders
- Lectures will be M 11-12, T&W 10-12, F 11-12.
- Check schedule on web for new times and rooms for some recitations (all are still on Thursday).
- Switching of recitations will be permitted if you have a conflict with another IAP activity.
- Contact your tutor about session scheduling
- Students working with Stephane Essame reassigned.
- Mastering Physics due this Wednesday at 10pm.
- Pset due this Friday at 11am.

Complete Description of Motion
- For an extended object, all of the equations learned last fall apply exactly without approximations to the motion of the center of mass.
- This is true whether or not an object is also rotating.
- The two motions (linear position of the center of mass and rotation around the center of mass) can be considered separately, except for kinetic energy where everything gets lumped into one equation.

Kinematics Variables
- Position x
- Velocity v
- Acceleration a
- Force F
- Mass M
- Momentum p
- Angle θ
- Angular velocity ω
- Angular acceleration α
- Torque τ
- Moment of Inertia I
- Angular Momentum L

\[ \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \]

Vector Nature of Angular Motion
- Most of the problems we will consider will involve only motion about a single axis, effectively 1-D
- 1-D is still a vector (up/down or left/right, for example).
- For the two directions of rotation, use clockwise (CW) and counter-clockwise (CCW).
- The choice of which one is positive is arbitrary but you need to be consistent throughout a problem.
- However, CW and CCW do not a really specify a unique direction, so where is the vector?

First Strange Feature of Angular Motion
- What is the vector associated with angular motion?
- The only unique direction in the problem is the axis, but again, there are two directions along the axis.
- The vector for any angular quantity (θ, ω, α, τ, J) points along the axis with the direction given by a right-hand-rule.
- Fingers curl in direction of ω, α, τ, J, thumb points in the direction of the vector
Kinematics of Angular Motion
- All equations are exactly identical to those for linear motion but with the variables renamed.
  - If $a$ is constant: $v = v_0 + at$ \hspace{1cm} $x = x_0 + v_0t + \frac{1}{2}at^2$
  - If $\alpha$ is constant: $\omega = \omega_0 + \alpha t$ \hspace{1cm} $\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$

Constraints/Connections between Linear and Angular Motion
- For all points on a rigid object and for any axis considered, the value of $\omega$ and $\alpha$ are identical.
- If a point $a$ distance $R$ from the axis of a rotating object, the path traveled and the angle are related by $S=R\theta$, which implies that $v=R\omega$, and $a=R\alpha$.
- Tangential acceleration of a point a distance $R$ from the axis of a rotating object is $R\alpha$.
- Radial acceleration of a point a distance $R$ from the axis of a rotating object is $v^2/R$.

Rolling Without Slipping
- For an object rolling without slipping, the point in contact with the ground has a speed of exactly zero so that it is not sliding along the ground.
- Use relative velocity concepts to show that the center of mass of the wheel must be moving with a velocity given by $v=R\omega$ and an acceleration $a=R\alpha$.
- The same constraints apply for a string on a pulley that is not slipping. In this case, $v$ and $a$ are for the end of the string and $\omega$ and $\alpha$ are for the pulley.

Torque
- How do you make something rotate? Very intuitive!
  - Larger force clearly gives more "twist".
  - Force needs to be in the right direction (perpendicular to a line to the axis is ideal).
  - The "twist" is bigger if the force is applied farther away from the axis (bigger lever arm).
- In math-speak: $\tau = \vec{r} \times \vec{F}$ \hspace{1cm} $|\tau| = ||\vec{F}||\sin(\phi)$

More Ways to Think of Torque
- Magnitude of the force times the component of the distance perpendicular to the force (aka lever arm).
- Magnitude of the radial distance times the component of the force perpendicular to the radius.
- Direction from Right-Hand-Rule for cross-products and can also be thought of as clockwise (CW) or counter-clockwise (CCW).
- For torque, gravity acts at the center of mass.

Conditions for Equilibrium
- Same as before: $\Sigma \vec{F} = 0$
  - It's totally irrelevant where the forces are applied to an object, only their direction and magnitude matters.
  - This gives one independent equation per dimension.
- Additional condition: $\Sigma \vec{r} = 0$
  - This is true for any axis. However, if all of the forces are in the same plane (the only type of problem we will consider in this class), you only get one additional independent equation by considering rotation.