

Let's look at the angular momentum of two particles, one sitting here and one over there.

And they are circling each other.

And we want to determine the angular momentum of a point on the ground right here underneath.

That's the whole point, s .

And here is the center.

So the circle has radius, R . And here we have a height, h .

and let's have the particles go counterclockwise.

Because then we can define our \hat{k} vector to go up.

And if these particles are going this way, then ω is also going to be going in the \hat{k} direction-- in the positive \hat{k} direction.

So we know that L equals r cross p .

So we need to determine where is r and what's going on with p .

So our r vector goes here from point s to our first particle, rs_1 .

And here is number two.

And we have rs_2 sitting over here.

And p , the momentum of the particles-- well, if these guys going in the counterclockwise direction, then the momentum of particle one is going to go into the board following the direction of the motion.

And here p_2 is going to come out of the board.

If we consider this equation here, we can already graphically determine what the answer is actually going to be.

And so we do r cross p .

So r cross p , which means L . Ls_1 is going to point in this direction.

Ls_1 .

And alternatively, L_{s2} is going to go this direction, which means if we add these two components-- and we do want the total angular momentum of the system of both particles.

So we have to add them.

We'll get here our L of the system.

So in order to calculate that, we need to calculate first the L_{s1} and then the L_{s2} .

And then we need to add them.

The crucial point here is to be careful with the coordinate system.

For the first particle, \hat{k} is going up.

And \hat{r} is going outwards.

And $\hat{\theta}$ is going into the board.

Then we can write up $L_{s1} = r_{s1} \times p_1$.

And we can express this vector here with \hat{h} and with \hat{r} .

So we have R in the \hat{r} direction plus h in the \hat{k} direction cross p .

And p is the mass of the particle times the angular velocity, the component.

That is $R \omega_z$.

And that goes in the $\hat{\theta}$ direction.

So we need to solve this cross product here.

And we're going to have $mR^2 \omega_z$.

And then we have $\hat{r} \times \hat{\theta}$.

That's $\hat{k} + h m R \omega_z$.

And then we have $\hat{k} \times \hat{\theta}$.

That's anti-cyclic.

And we get an \hat{r} .

And this is, by the way, an \hat{r}_1 .

And this is another \hat{r}_1 .

And we get a minus because it's anti-cyclic.

So we're going to turn this here into a minus.

And that is the angular momentum for our first particle.

Now we're going to do the same in an analogous way for particle number two.

But again, we need to look at the coordinate system.

That's very crucial here.

So \hat{k} still goes up.

\hat{r} now goes into the other direction.

And accordingly, $\hat{\theta}$ comes out of the board.

And then we can just write down our L_2 here as the same thing, $mR^2 \omega \hat{k}$.

And here we have minus $mR \omega \hat{r}_2$.

And we know from our setup here that \hat{r}_1 equals minus \hat{r}_2 .

\hat{r}_1 equals minus \hat{r}_2 , which means when we now put it all together, we want L equals L_1 plus L_2 .

We can tally this up.

And from this, we will see that the \hat{r} term actually falls out.

And we are left with 2 times the first term.

So we have $2mR^2 \omega \hat{k}$.

And that's exactly what we wanted.

We wanted an L pointing in the \hat{k} direction.

That's exactly this here.

And what you can also see is that the $2m$ here is the mass, the total mass of the system, namely two particles.

We could add another-- we could do the same exercise again for four points because we'll add two here.

Then we would get $4m$ here.

So this is the mass of the system.

And if we were to add many, many more points all along the rim here, we get to a solid ring, which means that either we know the total mass of the ring that we could stick in here.

Or we can integrate along the rim and then get to the total mass here.

And we'll always get to the same kind of form.