

The motion of objects in space is governed by the universal law of gravity.

So let's consider how this works.

We have two objects here, object one and two.

And that could be two planets or two asteroids or two white dwarfs, black holes, any kind of objects you can imagine.

And they are mutually attracted by each other due to gravitation.

So we have a gravitational force and we label this as F .

And this is the force on object one due to the interaction between the bodies one and two.

And here we have a force on object two due to the interaction between objects one and two.

And the objects are separated by a distance $r_{1,2}$.

Now, we want to derive the universal law of gravitation.

How are we going to go about that?

Well, Newton figured out a while ago that it is proportional to the masses of objects one and two.

So this one has a mass m_2 and this one has m_1 .

And it is also proportional to the square of the distance between the two objects.

Now, we need to do one more consideration and then we can derive this.

We need to actually-- well, first, we need to pick some kind of origin from where we are considering these two objects to be.

And this object here goes from the origin to there and we call this r_1 .

And then we have here r_2 , which also means that the distance here between object one and two is $r_{1,2}$.

And actually, we know from vector decomposition that $r_{1,2}$ equals r_2 minus r_1 .

So just this minus this gives us this distance here.

Now, if we want to write down the universal law of gravitation, there's a magnitude component to it and we also need a direction.

And we haven't yet chosen a coordinate system.

We could, of course, choose our usual way of placing the \hat{i} direction in the x-direction and the \hat{j} coordinate in the y-direction.

But when we deal with the universal law of gravitation, it's actually better to adopt a slightly different coordinate system.

Everything in space usually orbits one another so it's much better to think in a radial direction rather than just normal Cartesian coordinates.

And so in this case, we're going to choose an \hat{r} vector which gives us a radial direction and we're going to do this here.

So this is going to be our \hat{r} direction on object two.

And here, we have an \hat{r} direction 2, 1.

One And we're going to come back to the \hat{r} unit vectors later.

For now, we can just write here quickly down the definition for a unit factor.

So our $\hat{r}_{1,2}$ is, of course, the vector itself, $r_{1,2}$, divided over the magnitude of the vector.

We can write it like this.

And now we can write down the gravitational law.

So the force on object-- we're going to look at object two.

The force on object two due to the interaction between objects one and two is proportional to the mass of the two objects-- we already said that in the beginning-- and the square of the distance between the two objects.

But what about the direction?

The direction here, we're looking at object two.

We placed our \hat{r} unit vector to point down but the force is going in the opposite direction-- so in the negative

$\hat{r}_{1,2}$ direction.

So we have to add a minus here and then our $\hat{r}_{1,2}$.

And as it is the case with most of these laws, it has a proportionality constant and Newton called this capital G.

And G, as we know it today from experiment, is 6.67×10^{-11} .

And then in terms of units, we have Newton.

Force goes in Newton.

We have mass.

This is kilogram squared and we have meter squared.

So those are the units.

And if you plug those in, then the units of the whole equation will work out.

So let's quickly consider the force in object one to see what's happening over there.

So we have $F_{2,1}$ equals minus $G m_1 m_2$ over $r_{1,2}^2$.

And in terms of the unit vector, we now have $\hat{r}_{2,1}$ here going.

So again, this minus goes with this unit factor and that one is pointing here in the opposite direction than our force.

So that's all good.

But what we see from this one here-- and actually from our diagram already-- that $r_{1,2}$ equals minus $r_{2,1}$.

And so we see from this then that actually, Newton's third law, every action has an equal and opposite reaction, is true for this little setup here, as well, because the forces are of opposite direction and of equal magnitude.