

Let's consider the following problem.

Suppose we have an inclined plane.

And on this inclined plane, we have a block.

And down here at the bottom of the inclined plane is a little bumper.

It can be a spring of some type.

And what we have is the block is sliding down the inclined plane, bouncing off the bumper, and sliding up the inclined plane.

And what we'd like to do is find the acceleration of the block for both cases.

So when it's sliding down, let's do an analysis on that motion.

Well, the first thing we have to do is, again, identify our system.

In this case, just the block will be our system.

So we'll put in the block as our system.

And now we have to analyze the forces on the block as it's sliding down.

There is a friction with a coefficient μ_k .

And so we have the gravitational force, mg .

We have a normal force of the surface on the block pointing up.

And because the block is sliding down, remember that our friction force is opposing that motion.

Now when you have a free body force diagram, we have to decide how should we choose our coordinate systems.

What we want to encourage you is not to choose a coordinate system, unit vectors, horizontal and vertical, just because that's the way one always does it, but to use the constraints of the system to help you choose the coordinate system.

Now a technique for doing that is to draw what you think will be the acceleration on your sketch.

So I expect that a is going to go down the inclined plane.

That's taking into account that there is no motion perpendicular to the inclined plane.

Drawing the acceleration-- remember, acceleration is not a force-- but this gives me a clue for how I should choose my unit vectors.

I'll choose them, so that I have points down.

And so I essentially have a one dimensional motion.

And I'll choose \hat{j} pointing out.

Now I'll introduce an angle θ .

That's the same angle θ here.

And one has to be a little bit careful in doing these types of sketches.

But this is also the angle θ .

And now I've chosen a coordinate system, unit vectors.

I've indicated relevant angles.

And the rest of the problem is just vector decomposition in applying Newton's second law.

So we'll write down $F = ma$.

And I'm going to indicate a_d for we're talking about the acceleration downwards.

And we have two directions to analyze, the \hat{i} and the \hat{j} directions.

So in the direction tangent to the surface, we have a gravitational force, downward, $mg \sin \theta$.

We have the kinetic friction force opposing that motion.

And that's equal to ma_d , where a_d is the x component of the acceleration.

In the vertical direction, we have the normal force minus $mg \cos \theta$.

And here's where our choice of coordinate system helps us.

The constraint of the system is that a_y is 0 when we choose y to be the perpendicular direction.

So this side is 0.

Now I look at this, and I see a f and N .

And that's only two equations and three unknowns.

But now I have a for slope for my friction, which is that the kinetic friction is equal to a coefficient $\mu_k N$.

And I can see from this equation, that's $\mu_k mg \cos \theta$.

And now I'm in position in which to substitute my friction into this equation and solve for the acceleration.

Notice the mass will cancel.

And what I get is that the acceleration going down is just equal to $g \sin \theta - \mu_k \cos \theta$.

And so that's how we can analyze the acceleration when the block is sliding down.

Next, we want to analyze the acceleration when the block is going up.

Now when you have a case like that, it's sometimes easier, again, just to draw.

We'll still choose N .

Now, the interesting thing is, which way is the acceleration pointing?

Just because the block is going up, the acceleration is still pointing down.

How do you know that?

Well, we'll see that when we analyze the forces.

So we still have N . We have mg .

But now, the crucial point is-- this is going up-- the crucial point here is that the kinetic friction is always opposing the motion.

So when the block is sliding up, the kinetic friction is down.

And all that we have is a sign change here.

This is now plus.

If I choose my \hat{i} and \hat{j} in the same direction, notice both of these forces have a positive component in the \hat{i} direction.

So the acceleration will be down.

And rather than do all the algebra, all I'm doing is changing the sign.

And I get that a_{up} is equal to $g \sin \theta + \mu_k \cos \theta$.

And although it may seem slightly counter-intuitive that the acceleration as the block is going up in magnitude is bigger than the acceleration as the block is going down.

And that's because of the direction of the kinetic friction force.